UNIVERSITÀ
DEGLI STUDI DI TRIESTE

dipartimento di matematica e geoscienze

INdAM

Rendiconti's Jubilee - an INdAM day in Trieste

Università degli Studi di Trieste, 29 January 2018

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ABSTRACTS

Ugo Bruzzo (SISSA)

Title: Tannaka categories and fundamental groups

Abstract: Tannaka categories are categories that are equivalent to categories of representations of an algebraic group. M.V. Nori used this fact to define a fundamental group which is well suited to study varieties over fields of positive characteristic. The same idea can be used to introduce other generalizations of the fundamental group which "feel" some specific geometric properties.

Julián López-Gómez (Universidad Complutense, Madrid)

Title: Algebraic Invariants to detect the changes of the Topological Degree in Nonlinear Analysis

Abstract: Global Nonlinear Analysis is imperative to obtain global results in the context of nonlinear differential equations and hence, its number of applications to all areas of science is certainly huge. Most of these applications are based on the fact that the local index of some solution of a fixed point equation for a certain compact operator changes when some parameter of physical or mathematical interest varies, for as these local changes entail global bifurcation phenomena. Although the Schauder formula provides us with the local index in terms of a certain sum of algebraic multiplicities of eigenvalues, it is of no practical utility in applications. This talk describes the construction of a concept of Algebraic Multiplicity, easily computable in applications, which is an optimal invariant to detect any change of the local fixed point index.

Bibliography:

[1] J. López-Gómez, Global bifurcation for Fredholm operators, *Rend. Istit. Mat. Univ. Trieste* **48** (2016), 539-564.

[2] J. López-Gómez, *Spectral Theory and Nonlinear Functional Analysis*, Research Notes in Mathematics 426, CRC Press, 2001.

[3] J. López-Gómez and C. Mora-Corral, *Algebraic Multiplicities of Eigenvalues of Linear Operators*, Operator Theory, Advances and Applications 177, Birkhauser, 2007.

Daniele Faenzi (Université de Bourgogne, Dijon)

Title: Vector bundles and rational maps of the projective space

Abstract: A rational (self-)map f of the projective space P^n is defined by sending $x=(x_0,..,x_n)$ to $(f_0(x),..,f_n(x))$ where the f_i are homogeneous polynomials in x. The map f is actually defined away the "base locus", namely those x where all f_i vanish. The fibre at the point y is the preimage via f of y, and the degree of f is the number of points in the fibre of y (but not in the base locus), when y is "generic". How to compute the degree of f starting from the f_i? I will give an overview of what is known about this question and on how vector bundles can help classifying rational maps. I will also focus on the case when f is of "polar type" or a "Gauss map", namely when the f_i are the partial derivatives of a homogeneous polynomial. This contains a report on the Ph. D. thesis of R. Bignalet-Cazalet (U. Bourgogne).

Aldo Conca (Università di Genova)

Title: Koszul algebras and their syzygies

Abstract: Koszul algebras are certain algebras defined by quadratic relations; most quadratic algebras that arise naturally are in fact Koszul. There is no finite criterion for Koszulness, however, so it is interesting to study necessary or sufficiently conditions. I will explain what Koszul algebras are, and describe special features of their syzygies.

Gianluigi Rozza (SISSA, Trieste)

Title: Galerkin-RB-POD Reduced Order Methods: state of the art and perspectives with focus on parametric Computational Fluid Dynamics

Abstract: In this talk, we provide the state of the art of Reduced Order Methods (ROM) for parametric Partial Differential Equations (PDEs), and we focus on some perspectives in their current trends and developments, with a special interest in parametric problems arising in Computational Fluid Dynamics (CFD). Systems modelled by PDEs are depending by several complex parameters in need of being reduced, even before the computational phase in a pre-processing step, in order to reduce parameter space.
Efficient parametrizations (random inputs, geometry, physics) are very important to be able to properly address an offline-online decoupling of the computational procedures and to allow competitive computational performances. Current ROM developments in CFD include: a better use of stable high fidelity methods, considering also spectral element method, to enhance the quality of the reduced model too; more efficient sampling techniques to reduce the number of the basis functions, retained as snapshots, and the dimension of online systems; the improvements of the certification of accuracy based on residual based error bounds and stability factors, as well as the the guarantee of the stability of the approximation with proper space enrichment. For nonlinear systems, also the investigations on bifurcations of parametric solutions are crucial and they may be obtained thanks to a reduced eigenvalue analysis. All the previous aspects are very important in CFD problems to

be able to focus in real time on complex parametric industrial and biomedical flow problems, or even in a control flow setting, and to couple viscous flows -velocity, pressure, as well as thermal field - with a structural field or a porous medium, thus requiring also an efficient reduced parametric treatment of interfaces between different physics.
Model flow problems will focus on few benchmark cases in a time-dependent framework, as well as on simple fluid-structure interaction problems. Further examples of applications will be delivered concerning industrial shape optimization.