T×W Epistemic Modality

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ABSTRACT. So far, T×W frames have been employed to provide a semantics for a language of tense logic that includes a modal operator that expresses historical necessity. The operator is defined in terms of quantification over possible courses of events that satisfy a certain constraint, namely, that of being alike up to a given point. However, a modal operator can as well be defined without placing that constraint. This paper outlines a T×W logic where an operator of the latter kind is used to express the epistemic property of definiteness. Section 1 provides the theoretical background. Sections 2 and 3 set out the semantics. Sections 4 and 5 show, drawing on established results, that there is a sound and complete axiomatization of the logic outlined.

KEYWORDS: time, worlds, epistemic modality.

1.   Introduction

This paper originates from some reflections on future contingents. Among those who have attempted to provide a rigorous account of future contingents, there is a widespread tendency to think that the most appropriate formal semantics for

a tensed language involves branching time structures, that is, structures formed by a set of times and a tree-like partial order defined on the set. This inclination is fostered by two assumptions. One is that indeterminism entails *branching*, that is, the conception according to which there is a plurality of possible courses of events that overlap up to a certain point, the present. The other is that an adequate account of the semantic properties of future contingents hinges on the notion of *determinacy*, understood as truth in all possible courses of events. In a branching time structure, overlapping possible courses of events are represented as maximal linearly ordered subsets of times, and determinacy is expressed in terms of truth at a time relative to all possible courses of events that include that time[1].

However, both assumptions might be rejected. Against the first it may be argued that, at least on a plausible understanding of indeterminism, indeterminism does not entail branching. If determinism is understood as the claim that for any time, the state of the universe at that time is entailed by the state of the universe at previous times together with the laws of nature, and indeterminism is understood as the negation of that claim, then indeterminism is consistent with a conception according to which possible courses of events do not overlap. Possible futures may be conceived as parts of possible worlds that are wholly distinct, rather than branches that depart from a common trunk[2].

The second assumption may be questioned in at least two ways. In the first place, it may be argued that any account of future contingents centred on the notion of determinacy neglects a crucial distinction, namely, that between truth and determinate truth. Suppose that the following sentence is uttered now

(1) It will rain

It is at least consistent to claim that (1) may be true even though it is not determinately true, if it is true in the actual course of events but false in some other possible course of events. Secondly, it may be argued that some of the facts that the notion of determinacy is intended to capture in reality are epistemic facts,

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[1] The notion of branching time structure goes back to Kripke, see [9], pp. 27-29.

[2] Hoefer considers a definition of indeterminism along these lines, see [5]. Lewis argues against branching in [8], pp. 206-209. In [7] I discuss the claim that indeterminism entail branching.

hence that an account of them in terms of a formal representation of a state of knowledge is preferable to one that depends on unnecessary metaphysical assumptions. Consider the apparent difference between (1) and the following sentence

(2) Either it will rain or it will not rain.

This difference can be explained epistemically as follows: now we are not able to tell whether (1) is true because as far as we know (1) is true in some but not in all possible courses of events. By contrast, we can confidently assert (2), as (2) seems to be true in all possible courses of events[3].

If the two assumptions are rejected, no strong motivation remains for regarding branching time structures as a privileged formal tool to deal with the issue of future contingents. In particular, if the second assumption is rejected on the basis of considerations about the epistemic nature of facts such as that considered, there seems to be no reason to restrict attention to metaphysical interpretations of formal semantics. This paper explores one of the alternative routes. The model of time that will be outlined, *the grid model*, belongs to the family of T×W semantics, and the interpretation of it that will be considered is epistemic rather than metaphysical[4].

2. The grid model

Let Φ be the set of propositional variables. Our language will be the smallest set including Φ that is closed under composition by means of the propositional connectives and the operators G, H and D. Its semantics is based on the following definition.

DEFINITION 1. *Let T and W be sets. A $\mathscr{G}$-frame is a pair $\langle \{T_w, <_w\}_{w \in W}, \approx \rangle$ that satisfies the following conditions.*

   1. For any $w \in W$, $T_w \subseteq T$. For any $w, w' \in W$ such that $w \neq w'$, $T_w \cap T_{w'} = \emptyset$.

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[3] In [7] I argue for the distinction between truth and determinate truth.
[4] The original formulation of T×W semantics is given by Thomason in [10]. The epistemic interpretation that will be considered develops a suggestion that I advanced in [6].
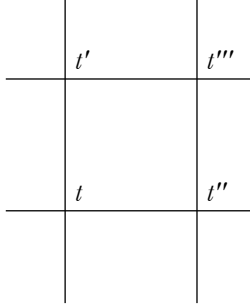
FIG. 1: *The grid of worlds and instants*

2. *For any $w \in W$, $<_w$ is a linear order on T_w. A relation $<$ on T is defined accordingly: for any $t, t' \in T$, $t < t'$ iff there is a w such that $t, t' \in T_w$ and $t <_w t'$.*

3. *$\approx$ is an equivalence relation on T such that (a) for any $t \in T$ and $w \in W$, there is a unique t' such that $t' \in T_w$ and $t \approx t'$, (b) if $t, t' \in T_w, t'', t''' \in T_{w'}, t \approx t'', t' \approx t'''$ and $t <_w t'$, then $t'' <_{w'} t'''$.*

From now on, Dn will abbreviate 'definition n', and D$n.m$ will abbreviate 'clause m of definition n'. The members of T are called times. The members of W are called worlds. So from D1.1 and D1.2 it turns out that worlds are linearly ordered disjoint sets of times. This means that times are world-relative temporal units, in that each time belongs at most to one world. The relation $\approx$ specified in D1.3, by contrast, expresses the trans-world relation of "being at the same time", so induces a partition of times that is orthogonal to their chaining into worlds. To make this clear, the equivalence classes of times determined by $\approx$ may be called "instants", following the terminology adopted by Belnap, Perloff and Xu in [1]. In figure 1, worlds are represented as straight vertical lines that run parallel, whereas instants are represented as straight horizontal lines that cut across them: for example, t and t' belong to the same world, while t and t'' belong to the same instant.

DEFINITION 2. *A $\mathscr{G}$-structure is a triple $\langle \{T_w, <_w\}_{w \in W}, \approx, V \rangle$, where $\langle \{T_w, <_w\}_{w \in W}, \approx \rangle$ is a $\mathscr{G}$-frame and V is a function that assigns a truth-value to each*

formula α for any time t in the following way:

1. $V_t(\alpha) \in \{1,0\}$ *for* $\alpha \in \Phi$.

2. $V_t(\sim \alpha) = 1$ *iff* $V_t(\alpha) = 0$.

3. $V_t(\alpha \supset \beta) = 1$ *iff* $V_t(\alpha) = 0$ *or* $V_t(\beta) = 1$.

4. $V_t(G\alpha) = 1$ *iff for every* t' *such that* $t < t'$, $V_{t'}(\alpha) = 1$.

5. $V_t(H\alpha) = 1$ *iff for every* t' *such that* $t' < t$, $V_{t'}(\alpha) = 1$.

6. $V_t(D\alpha) = 1$ *iff for every* t' *such that* $t \approx t'$, $V_{t'}(\alpha) = 1$.

D2.1-D2.3 are standard. D2.4 and D2.5 specify the meaning of G and H, read as 'henceforth' and 'hitherto'. D2.6 characterizes D, read as 'definitely'. D differs from G and H in a way that is easy to grasp visually. G and H ask you to move along the vertical axis and go up or down on the same world, while D asks you to move along the horizontal axis and go left and right on the same instant[5].

Other symbols may be added to the language on the basis of D2. $\land$ and $\lor$ depend on $\sim$ and $\supset$ in the usual way. Two operators F and P may be defined in terms of G and H as follows: $F\alpha \equiv \sim G \sim \alpha$ and $P\alpha \equiv \sim H \sim \alpha$. Similarly, an operator C may be defined in terms of D as follows: $C\alpha \equiv \sim D \sim \alpha$. Truth in a structure and validity are defined in the standard way:

DEFINITION 3. α *is true in a $\mathscr{G}$-structure iff for any t,* $V_t(\alpha) = 1$.

DEFINITION 4. α *is valid, that is,* $\models \alpha$, *iff* α *is true in all $\mathscr{G}$-structures.*

The semantics outlined is a kind of T×W semantics. In particular, the version of T×W semantics that best suits the present purposes is that provided by Kutschera in [11]. Kutschera defines a STW frame as a triple $\langle \{T_w, <_w\}_{w \in W}, \approx, \sim \rangle$, where the first two terms satisfy the conditions specified in D1 and $\sim$ is an equivalence relation that differs from $\approx$ in that it is designed to express historical necessity. So $\mathscr{G}$-frames are nothing but STW frames without that relation[6].

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[5] A modal operator defined in terms of unrestricted quantification over worlds, like D, was first considered by Di Maio and Zanardo in [2].

[6] The letter S in STW stands for 'separated', to distinguish STW frames from standard T×W frames as defined by Thomason in [10]. In [11], Kutschera shows that for every T×W structure there is an equivalent STW structure, and vice versa, see p. 243.

3. The epistemic interpretation

On the interpretation of the grid model that will be considered, times designate epistemically possible global states of affairs, and worlds are understood as epistemically possible courses of events. The underlying thought is that, for every sentence 'p' such that we are not in a position to know that p, there are at least two worlds: one in which p and one in which it is not the case that p. For example, today we are not in a position to know whether it will rain tomorrow. So there are at least two worlds: one in which it rains tomorrow and one in which it doesn't.

The use of the expression 'in a position to know' presupposes that a meaningful distinction can be drawn between knowing that p and being in a position to know that p. Being in a position to know that p, like knowing that p, is factive: if one is in a position to know that p, then it is true that p. But the two states are not exactly the same. While knowing that p entails being in a position to know that p, being in a position to know that p does not entail knowing that p: one may be in a position to know that p without knowing that p, just like one may fail to see something that is in front of one's eyes[7].

Note that the differences between epistemically possible courses of events are not limited to the future. For example, we are not in a position to know whether the number of cats that slept inside the Colosseum on September 4th 1971 is even or odd. So there are at least two worlds: one in which that number is even, the other in which that number is odd. The same goes for the present. For example, we don't know the exact location of a certain whale that is now swimming in the ocean, so we are not able to discriminate between times that differ as to the location of that whale. The absence of a unique present time is a key feature of the grid model. In figure 1 there is no point that indicates where you are. The reason is that you don't know exactly where you are, in that you don't know which of the two worlds is your world. What allows you to locate yourself on the diagram is a line rather than a point, that is, an instant.

On the epistemic interpretation, D expresses truth in all epistemically possi-

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[7] Williamson provides a characterization of the distinction along these lines, see [12], p. 95. In any case, nothing substantial will depend on this distinction. The underlying thought of the epistemic interpretation could equally be rephrased as follows: for every sentence 'p' such that we don't know that p, there are at least two worlds: one in which p and one in which it is not the case that p.

ble courses of events. To say that it is definitely the case that p is to say that one is in a position to know that p. For example, the apparent difference between (1) and (2) may be explained in terms of definiteness. Consider figure 1. If p is true at t' but false at t''', then DFp is false at t, while $D(Fp \lor \sim Fp)$ is true at t.

The operator C is construed accordingly. To say that one is in a position to know that p is to say that every epistemically possible course of events is such that p. Therefore, if one is not in a position to know that it is not the case that p, then one is not in a position to exclude that p, that is, some epistemically possible course of events is such that p.

4. Axiomatization

T×W logic has been shown to be complete under two axiomatizations. One is the finite axiomatization adopted by Kutschera in [11], which includes the irreflexivity rule introduced by Gabbay in [4]. The other is the infinite axiomatization adopted by Di Maio and Zanardo in [3], which is free from that rule. The system outlined here follows Kutschera, for the completeness proof is simpler with the irreflexivity rule. But a similar system could be constructed in terms of the other axiomatization.

Let S be a system whose axioms include the standard propositional axioms and the following:

A1 $G(\alpha \supset \beta) \supset (G\alpha \supset G\beta)$

A2 $H(\alpha \supset \beta) \supset (H\alpha \supset H\beta)$

A3 $\alpha \supset HF\alpha$

A4 $\alpha \supset GP\alpha$

A5 $G\alpha \supset GG\alpha$

A6 $F\alpha \supset G(F\alpha \lor \alpha \lor P\alpha)$

A7 $P\alpha \supset H(F\alpha \lor \alpha \lor P\alpha)$

A8 $D\alpha \supset \alpha$

A9 $D(\alpha \supset \beta) \supset (D\alpha \supset D\beta)$

A10 $C\alpha \supset DC\alpha$

A11 $DG\alpha \supset GD\alpha$

A12 $DH\alpha \supset HD\alpha$

A13 $FD\alpha \supset DF\alpha$

A14 $PD\alpha \supset DP\alpha$

A1-A7 are standard axioms of linear tense logic. A1-A2 state that distribution holds for G and H. A3-A4 ensure that G and H depend on accessibility relations that are converse to each other. A5 expresses the transitivity of $<$. A6 rules out branching to the future, while A7 rules out branching to the past.

A8-A10 characterize D as a modal operator. A8 expresses a platitude, as it amounts to saying that being in a position to know is factive. A9 is easily justified. If one is in a position to know that if p then q and one is in a position to know that p, then one must be in a position to know that q. For all that is needed to get to the conclusion that q is to apply a valid rule of inference. A10 entails that if for all one knows it could be the case that p, then one is in a position to know that for all one knows it could be the case that p. This is quite plausible. Suppose that one is not in a position to know that it is not the case that p. Then, presumably, the negation of p does not hold in all possible courses of events in virtue of some logical principle, and one is in a position to know that.

A11-A14 state a connection between G, H, F and P on the one hand, and D on the other. According to A11, if it is knowable that from now on it will be the case that p, then from now on it will be knowable that p. This is acceptable if one thinks that the antecedent of the conditional is satisfied only for those truths that hold at any time. For example, it is true at any time in every epistemically possible course of events that if it rains then it rains. Thus, it is knowable that from now on if it rains then it rains. But if so then the consequent is satisfied, that is, from now on it will be knowable that if it rains then it rains. The motivation for A12-A14 is similar.

Let $\vdash$ stand for derivability in S. The rules of inference of S are the following:

R1 If $\vdash \alpha \supset \beta$ and $\vdash \alpha$, then $\vdash \beta$

R2 If $\vdash \alpha$, then $\vdash G\alpha$ and $\vdash H\alpha$

R3 If $\vdash \alpha$, then $\vdash D\alpha$

R4 If $\vdash D(\sim p \wedge Gp) \supset \alpha$, then $\vdash \alpha$, where p is a propositional variable that does not occur in α

R1 is *modus ponens*, R2 is temporal generalization, while R3 amounts to the rule of necessitation. R4 is the version of Gabbay's irreflexivity rule used by Kutschera.

5. Soundness and Completeness

S is sound. It is straightforward to verify that A1-A14 are valid and R1-R4 preserve validity. The completeness of S can be proved through the method used by Kutschera for a system called TW. Kutschera defines STW systems as sets of maximal consistent sets of formulas endowed with a relational structure, and shows that STW systems induce STW structures. Thus, in order to prove that TW is complete it suffices to show that for every formula that is not a theorem of TW there is a STW system that includes its negation, for that in turn entails the existence of a STW structure in which the formula is not true at some time. Here a proof will be provided to the effect that STW systems induce $\mathscr{G}$-structures. So the completeness of S will be obtained in the same way, using von Kutschera's result about the existence of a STW system.

Let us grant Kutschera's definition of STW systems. To abbreviate, 'mcs' will stand for 'maximal consistent set of formulas'. The relation R_G is defined as follows: if S and S' are mcss, SR_GS' iff $G(S) \subseteq S'$, where $G(S) = \{\alpha : G\alpha \in S\}$. The relations R_H and R_D are defined in similar way. If S and S' are mcss, SR_HS' iff $H(S) \subseteq S'$, where $H(S) = \{\alpha : H\alpha \in S\}$. If S and S' are mcss, SR_DS' iff $D(S) \subseteq S'$, where $D(S) = \{\alpha : D\alpha \in S\}$. R_G and R_H are transitive, while R_D is an equivalence relation.

DEFINITION 5. *A STW system is a pair* $\langle \{S_t\}_{t \in T}, \{T_w\}_{w \in W} \rangle$ *defined as follows.*

1. *W is set of indices.*

2. *The sets T_w are disjoint, and T is the union of them.*

3. *For every $t \in T$, S_t is a mcs. Each $t \in T$ has its own mcs, so if $t \neq t'$ then $S_t \neq S_{t'}$.*

4. *For every $w \in W$ and $t \in T_w$, if $F\alpha \in S_t$ then there is a $t' \in T_w$ such that $S_t R_G S_{t'}$ and $\alpha \in S_{t'}$. The same goes for P and R_H. The case of C and R_D is similar, but without the condition that $t' \in T_w$.*

5. *For every $t, t' \in T_w$, either $S_t = S_{t'}$ or $S_t R_G S_{t'}$ or $S_{t'} R_G S_t$.*

6. *For every $t \in T$ and for some propositional variable p, $D(\sim p \wedge Gp) \in S_t$.*

7. *For every $w, w' \in W$ and every $t \in T_w$, there is a $t' \in T_{w'}$ such that $S_t R_D S_{t'}$.*

Let it be granted that $t <_w t'$ iff $t, t' \in T_w$ and $S_t R_G S_{t'}$, and that $t \approx t'$ iff $S_t R_D S_{t'}$. Now it will be shown that for every STW system there is a correspondent $\mathscr{G}$-structure.

THEOREM 1. *If $\langle \{S_t\}_{t \in T}, \{T_w\}_{w \in W} \rangle$ is a STW system, then $\langle \{T_w, <_w\}_{w \in W}, \approx \rangle$ is a $\mathscr{G}$-frame.*

PROOF. D1.1 follows from D5.2. D1.2 follows from D5.5. To see that D1.3 is satisfied, consider condition (a) first. The existence of t' is entailed by D5.7. The uniqueness of t' is shown as follows. Suppose that $t, t' \in T_w$ and $t \approx t'$. From D5.5 we get that either $S_t = S_{t'}$ or $S_t R_G S_{t'}$ or $S_{t'} R_G S_t$. But the second disjunct cannot hold, because from D5.6 we get that $Gp \in S_t$, hence that $p \in S_{t'}$. Since we also have that $D \sim p \in S_t$, hence that $\sim p \in S_{t'}$ because $t \approx t'$, we get that both $p \in S_{t'}$ and $\sim p \in S_{t'}$, which contradicts D5.3. A similar reasoning shows that the third disjunct cannot hold. Therefore, $S_t = S_{t'}$.

Now consider condition (b). Suppose that $t, t' \in T_w$, $t'', t''' \in T_{w'}$, $S_t R_D S_{t''}$, $S_{t'} R_D S_{t'''}$ and $S_t R_G S_{t'}$. From D5.5 we get that either $S_{t''} = S_{t'''}$ or $S_{t''} R_G S_{t'''}$ or $S_{t'''} R_G S_{t''}$. But the first disjunct cannot hold. D5.6 entails that $D(\sim p \wedge Gp) \in S_t$, hence that $\sim p \in S_{t''}$. Since we also have that $DGp \in S_t$, A11 entails that $GDp \in S_t$, hence that $Dp \in S_{t'}$ and consequently that $p \in S_{t'''}$. Therefore, the first disjunct contradicts D5.3. The third disjunct leads to a similar conclusion. For D5.6 entails that $D(\sim p \wedge Gp) \in S_t$, hence that $\sim p \in S_{t''}$. Since D5.6, in combination with A5, also entails that $DGGp \in S_t$, by A11 we get that $GDGp \in S_t$, and consequently that $DGp \in S_{t'}$. Since $S_{t'} R_D S_{t'''}$, it follows that $Gp \in S_{t'''}$. So if it were the case that $S_{t'''} R_G S_{t''}$, we would get that $p \in S_{t''}$. Therefore, $S_{t''} R_G S_{t'''}$. $\square$

THEOREM 2. *For each STW system $\langle \{S_t\}_{t \in T}, \{T_w\}_{w \in W} \rangle$ there is a $\mathscr{G}$-structure $\langle \{T_w, <_w\}_{w \in W}, \approx, V \rangle$ such that, for every α, $V_t(\alpha) = 1$ iff $\alpha \in S_t$.*

PROOF. Let $\langle \{S_t\}_{t \in T}, \{T_w\}_{w \in W} \rangle$ be a STW system. Theorem 1 entails that $\langle \{T_w, <_w\}_{w \in W}, \approx \rangle$ is a $\mathscr{G}$-frame. A function V can be defined on the frame in accordance with D2, assuming that, for each α in Φ, $V_t(\alpha) = 1$ iff $\alpha \in S_t$. This way it can be shown by induction on the complexity of α that for every α, $V_t(\alpha) = 1$ iff $\alpha \in S_t$. The case of $\sim \alpha$ and $\alpha \supset \beta$ is trivial. Consider the case of $G\alpha$. Let us assume that $V_t(\alpha) = 1$ iff $\alpha \in S_t$, and suppose that $V_t(G\alpha) = 1$. From D2.4 we get that for t' such that $t < t'$, $V_{t'}(\alpha) = 1$. Since $t < t'$ iff $S_t R_G S_{t'}$, $G\alpha \in S_t$ if $\alpha \in S_{t'}$. So $G\alpha \in S_t$. Now suppose that $G\alpha \in S_t$. Since $t < t'$ iff $S_t R_G S_{t'}$, for every t' such that $t < t'$ we get that $\alpha \in S_{t'}$, hence that $V_{t'}(\alpha) = 1$. So D2.4 entails that $V_t(G\alpha) = 1$. The case of $H\alpha$ and $D\alpha$ is similar. Therefore, $\langle \{T_w, <_w\}_{w \in W}, \approx, V \rangle$ is a $\mathscr{G}$-structure such that, for every α, $V_t(\alpha) = 1$ iff $\alpha \in S_t$. □

Kutschera proves that if a formula is not a theorem of TW, there is a STW system $\langle \{S_t\}_{t \in T}, \{T_w\}_{w \in W} \rangle$ such that for some $t \in T$, the negation of the formula belongs to S_t. A similar result holds for S, that is,

THEOREM 3. *If it is not the case that $\vdash \alpha$, then there exists a STW system $\langle \{S_t\}_{t \in T}, \{T_w\}_{w \in W} \rangle$ such that for some $t \in T$, $\sim \alpha \in S_t$.*

PROOF. In [11], pp. 246-247, Kutschera shows how theorem 3 can be proved in two steps. First, Gabbay's irreflexivity lemma can be used to show that if it is not the case that $\vdash \alpha$, then there is a set $\{S_t\}_{t \in T}$ that satisfies certain conditions and a t_0 such that $\sim \alpha \in S_{t_0}$ (theorem 4.1). S is like TW in this respect, as it includes the rule R4, which is required by the proof. Second, a STW system can be constructed from $\{S_t\}_{t \in T}$ by defining a set of $\{T_w\}_{w \in W}$ (theorem 4.2). Again, S is like TW in this respect, as it includes the axioms used in the proof. □

THEOREM 4. *If $\models \alpha$ then $\vdash \alpha$.*

PROOF. From theorems 2 and 3 it follows that if is not the case that $\vdash \alpha$, then there is a $\mathscr{G}$-structure such that $V_t(\alpha) = 0$ at some t. □

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