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On the Relational Constitution of Cosmic Spacetime

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ABSTRACT. Two important principles are placed at the foundations of standard cosmology: the *Cosmological Principle* (CP) and *Weyl's Principle* or *Postulate* (WP). The former can be found in all cosmology textbooks; the latter is often completely neglected. In this paper I attempt an inquiry on the nature of cosmic spacetime when standard cosmological models are approached by giving a special prominence to WP.[*]

1. Some remarks on Weyl's Principle

In Weyl's 1926 formulation[1] – referring to de Sitter universe which he considered at the time to be the most satisfying cosmological model – WP states:

> The worldlines of the stars form a sheaf, which rises in a given direction from the infinitely distant past, and spreads out over the hyperboloid in the direction of the future, getting broader and broader. (quoted in Goenner 2001, p. 121)

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[*] Many thanks to Jenny McCord for having improved my English.

[1] For historical aspects see Bergia (1991), Bergia and Mazzoni (1999), Ehlers (2009), Goenner (2001), Kerszberg (1986), Rugh and Zinkernagel (2011).

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Weyl suggests that the distribution of stars could be described by a congruence of non-intersecting timelike worldlines (a family of non-crossing curves), diverging (the universe is expanding) from a common point in the past. Furthermore, worldlines are thought to have been *causally interconnected* since their origin (Weyl 1930, p. 938).[2]

Nowadays, clusters of galaxies (or even clusters of clusters) are taken as the elementary constituents of the expanding universe because these giant agglomerates of matter follow the Hubble expansion pattern quite closely. Although clusters form a discrete set, one can extend it to a *continuum* by a smooth-fluid approximation. The idea is that one averages the speed of matter in a given large-scale region of the universe and assigns that speed and the mass of that region to a fictitious entity called a *fundamental particle* (one can imagine it placed at the center of mass of that region). Fundamental particles are *freely falling* insofar as their motions are affected by no forces except gravity and inertia. These material particles, when regarded as mere geometric points, constitute the kinematic *substratum* of the model. Each point is crossed by only one worldline. A reference frame is attached to each particle so that all matter of that region is at rest relative to that frame. In this way, a sort of global *comoving* (i.e. moving with the expanding motion of matter) reference frame system is defined.

The highly streamlined *large scale* motions of fundamental particles (no randomness, no vorticity, no collision, *except* at a singular point, the common “origin” in the past) (see fig. 1a) provide *natural synchrony calibration* for all events (the intersection theoretically defines the zero of time).[3] This guarantees that spacetime is globally resolved into space *and* time, i.e. that it can be foliated in a sequence of “space slices”, orthogonal to the bundle, whose succession instantiates the flow of what is called *cosmic time*.

Substratum is usually thought of as a kind of perfectly continuous background, a reference frame in uniform expansion. However, as each one of its points is an entity ideally “containing” all the matter present in a given cosmic region, substratum can be considered, at a “large-scale level of abstraction”, as an entity *in its own right*, whose particles are its real “atomic” (indivisible) material constituents.

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[2] The “*infinitely* distant past”, obviously clashing with the big bang theory, is negligible. On the other hand, Weyl himself specified that “all beginnings are obscure” (Weyl 1922, p. 10).

[3] Note that *small*-scales objects (galaxies, planets, etc.) have chaotic *peculiar* motions (see fig. 1b). Nonetheless, their very low velocities (less than one-thousandth of the velocity of light c) make them negligible when compared to large-scale *recessional* velocities of clusters (comparable to c).

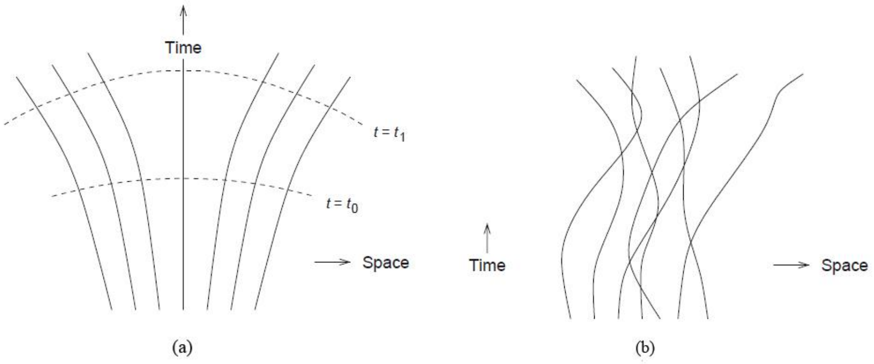

Fig. 1: (a) Systematic large-scales motions of clusters; (b) arbitrary motions of small-scales objects (from Narlikar 2010, p. 230).

2. The two approaches at the foundations of standard models

Standard cosmological models, the best description of the large-scale structure of our universe, are given by $< M, g_{ab}, T_{ab} >$, with M being the spacetime manifold, g_{ab} the so-called *Friedmann-Lemaître-Robertson-Walker* (FLRW) *metric*, and T_{ab} the stress-energy tensor representing the material contents of the universe (in the form of dust). This metric can be derived in two ways.

In the first and most often used way, it is derived just from CP.[4] Spacetime is split up by imposing homogeneity and isotropy: isotropy guarantees that worldlines are orthogonal to each spatial hypersurface (Misner et al. 1973, p. 714); the existence of cosmic time is a corollary of homogeneity (Rindler 2006, p. 359). WP is not explicitly mentioned.

In the second way – adopted, for example, by Bondi (1960), Narlikar (2010), Pauri (1995), Raychaudhuri (1979) – WP assumes a remarkably more elevated status: it is introduced first, it allows the definition of cosmic time and then the spacetime foliation. CP simply intervenes later, imposing homogeneity and isotropy on spatial hypersurfaces.

However, WP is *actually* always tacitly assumed (mathematically disguised in the notion of isotropy[5]) in the first way as well (Rugh and

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[4] CP states that the universe is spatially homogeneous and isotropic on large scales.

[5] For instance, see Wald (1984, p. 92).

Zinkernagel 2011).[6] The reason for this assumption is that WP is *necessary* for a physically well-defined notion of cosmic time. This importantly means that: “WP […] is a precondition for the CP; the former can be satisfied without the latter being satisfied but not vice versa” (2011, p. 417).

In this sense, the WP-based approach ought to be regarded as more fundamental than the CP-based one. Before deducing some consequences from this priority, a short premise is in order.

3. Deductive and constructive axiomatizations

A *deductive* axiomatic approach approximately “begins with a set of postulates concerning the existence of high level structures and/or principles and then proceeds by logical deduction to lower level phenomena which may be directly confronted by experiment” (Majer and Schmidt 1994, p. 17). Spacetime of General Relativity (GR) is usually introduced by this *top-down* way with formulae like: “spacetime *is* a 4-dimensional differentiable manifold… endowed with a semi-Riemannian metric…”.[7] Higher level spatiotemporal structure is therefore defined *from the outset* and assumed to be primitive, with a unifying explanatory role with respect to the lower level structures (affine, projective, conformal) governing physical behaviour of light and particles.

The *constructive* (or *inductive*) axiomatic approach is the “reverse” of the deductive one: “The constructive axioms deal with directly observable phenomena at as low a level as possible. The aim is to formulate axioms which may be directly confronted by experiment, and then deduce from these low level axioms the existence of higher level structures” (Coleman and Korté 1994, p. 68).

The problem of *deriving* the Riemannian “choice” by physically motivated axioms, rather than *postulating* it at the outset, was called by Castagnino (1971) the *inverse problem* of GR. In 1972, J. Ehlers, F. Pirani and A. Schild developed the most influential constructive approach to GR spacetime, called *EPS approach*. Starting from an initial structureless set of point-events, and using only freely falling particles and light rays and a small set of constructive axioms experimentally verified, they were able to build up, step by step, all

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[6] Also Pauri (1991, p. 334) hints at this thesis.

[7] For an example see Friedman (1983, p. 32).

GR spatiotemporal structures until reducing them to the desired pseudo-Riemannian metric.

CP-based and WP-based approaches may now be roughly seen, respectively, as examples of deductive and constructive methodologies. In the former, homogeneity and isotropy select the FLRW metric from a general semi-Riemannian metric given from the outset. In the latter, instead, we have an "inverse movement" *à la* Castagnino. Pauri explicitly states: "Matters are turned around with respect to the standard approach: a geodesic is a geodesic of some metric; here a particular geodesic structure is assumed in order to *construct* a metric having certain desired properties" (1991, p. 320).[8]

4. Constructive axiomatization and relationism

EPS procedure complies with a form of relationism. EPS ontology, indeed, consists only of particles and light rays, whereas spacetime has been ontologically demoted to their "by-product".

Ehlers explains:

> It has been shown that on the basis of simple facts the spacetime geometry of GR can be constructed without resorting to concepts or theorems of theories which presuppose such a geometry […] Only concepts by which relations between events, particles and light rays are describable have been introduced. This fully agrees with Leibniz's position of viewing space and time not as objects but rather as sets of spatial or temporal relations among things. (quoted in Jammer 1993, p. 229)

In the cosmological context, the result is supposedly the same: the pure adoption of WP, with its assigning an ontological primacy to fundamental particles, involves an important element of *relationality* in the conceptual foundations of FLRW spacetime.

Whitrow already stressed this implication: "The three-dimensional spatial cross-section is determined solely by the fundamental particles, i.e. it is a relational space and not an absolute space with an independent existence of its own" (1980, p. 292).[9] Pauri reached a similar conclusion as well: "The uni-

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[8] I am not claiming that this inverse approach should necessarily be that of the EPS, but that they could be closely related. On the other hand, EPS' and WP's particles are conceptually very similar.

[9] For a deeper analysis of Whitrow's view see Macchia (2011).

versal ‘substratum’ is defined by a specific structure of *virtual* (not in the quantum-mechanical sense!) trajectories of *fundamental* particles which *relationally constitute* spacetime” (1991, p. 319).

Therefore, the nature of spacetime is supervenient on the substratum. Each spacetime point is identified with a fundamental particle. Hence substratum can be thought of as a space-*constituting* rather than as a space-*filling* set of particles (as happens in the CP-based approach).

But this conclusion is not so general.

5. Space and substratum in de Sitter models

Let us take a look at de Sitter *empty* models, characterized by a null stress-energy tensor and a non-null cosmological constant Λ. In these models (fig. 2), unlike FLRW ones, there is no unique choice of congruence of worldlines representing the average motion of matter. As Rindler (2006, p. 400) remarks, non-empty models with different substrata have different spacetimes, but empty models with different substrata have identical spacetimes “since in the absence of density the spacetime is unaffected by the substratum” (p. 401). Accordingly, in de Sitter cases, choosing between two different congruences means choosing between models based on the same spatiotemporal background.

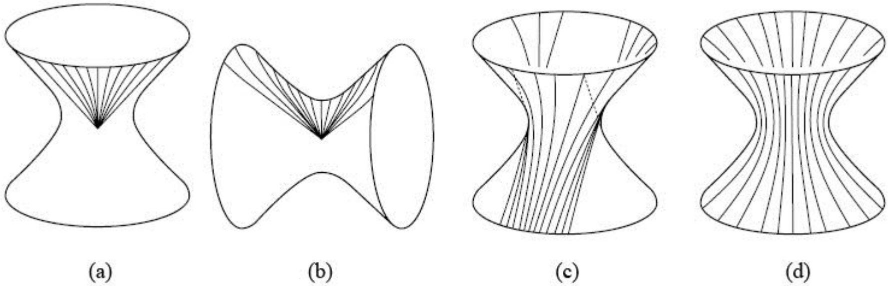

Fig. 2: Models (a), (c) (called *de Sitter universe*), and (d) share the *de Sitter spacetime* (the unique spacetime satisfying Einstein vacuum equations with $\Lambda > 0$); model (b) is based on the *anti-de Sitter spacetime* (with $\Lambda < 0$). Two spatial dimensions are suppressed (from Rindler 2006, p. 399).

Furthermore, models (a), (b), (c) in fig. 2 are *non-maximal*, in the sense of "there being more space than substratum" (*ibid*.), whereas model (d) is *maximal* insofar as substratum covers space completely.

It follows that only in maximal models may one have a complete identification between space and substratum, thence a relational constitution of the former. In de Sitter non-maximal models, instead, this fails because fundamental particles are not able to constitute the *entire* spacetime insofar as substratum is only *contained* in the spacetime (obtained as a solution of Einstein equations). Thereby a problem arises of *underdetermination* of spacetime by substratum. At most, different substrata may constitute *portions* (sometimes overlapping) of the de Sitter hyperboloid.

In epistemological terms, this fact highlights a well known necessity: in order to have a *universe model*, both an idealized substratum representing real matter and a geometrically defined metric (solution of Einstein equations) are required.[10] However, it is not at all clear to what extent that metric represents *real* spacetime. In ontological terms, those portions of de Sitter metric not covered by substratum then remain problematic. Are they only *surplus* structures, i.e. mathematical structures that do not refer to anything physically real?[11]

I do not think so. Even if one has more space than substratum *only* in *empty* models, the point is that these models are not *completely* empty: no ordinary matter is present, but Λ plays a fundamental role – even if its physical status is still unclear. I agree with Baker (2005): Λ's repulsion is not to be regarded as a sort of *anti*-gravitational force in the ordinary sense because, even if it influences the metric structure, it does *not* depend on matter as its source. So Λ, providing spacetime with an amount of curvature not entirely created by matter, may be considered as evidence in favor of a substantivalist ontology.

Furthermore, in de Sitter models, fundamental particles must be *test* particles, namely ideal (almost) massless particles (whose other physical properties are negligible) which do not represent anything but themselves, contrary to what happens in FLRW's cases. So, as Pauri reminds us, they are not the best objects upon which to found an ontology: "The notion of test body, already problematic if taken *a posteriori* within a physical theory […], is certainly

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[10] See Ellis and Williams (2000, p. 163).

[11] Weyl himself faced this problem speaking about a *complete cosmology* obtained with both the WP's assumptions and the "topological" assumption "concerning the question which part of the hyperboloid corresponds to the real world" (1930, p. 936).

even more problematic if taken *a priori*, as a constitutive element of the spacetime ontology" (1995, p. 449; my English translation).

In conclusion, it is quite difficult to figure out a constructive/relational approach in de Sitter models. The most plausible picture is instead given by a "substantival" spacetime: a background metric showing its expanding state as soon as a test particle is thrown into it. In short, no entity, apart from spacetime itself, is *ontologically* necessary. Test particles' role appears, in a sense, merely *epistemical*.

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