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Giovanni Vacca's contributions to the Historiography of Logic

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ABSTRACT. We intend to illustrate Vacca's contributions to the history of logic; analyse his historiographic position, making evident his merits and limits, and the epistemological assumptions to which it was anchored; and compare the work of Vacca with that of Federigo Enriques, author of the volume *Per la storia della logica*.

1. The history of logic in the *Formulaire*

Mathematical logic is a discipline with a relatively recent history, and yet as early as the end of the nineteenth century the first contributions to its historiography were being made by scholars working in Turin, including Giovanni Vacca (1872-1953). A highly cultured mathematician from Genoa, assistant to Giuseppe Peano from 1897 to 1905, polyglot with solid knowledge of both classical and modern languages, and refined bibliophile, Vacca was an esteemed historian of the sciences in his days. Author of a dozen articles on the history of logic,[1] his is the merit of having precisely identified the origin and

[1] Vacca, G.: "A.N. Whitehead, Treatise on Universal Algebra", *Rivista di Matematica* (*RdM*), 6, 1899, p. 101-104; "Sui manoscritti inediti di Leibniz", *Boll. Bibl. Sto. Sci. Mat. (Lo-*

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the key developments of the discipline, discovered through a careful examination of printed as well as manuscript sources. His interest in the history of logic ran like a thread through his entire professional career, and was a leitmotiv in his correspondence with Peano, G. Vailati, L. Couturat, M. Cantor and G. Loria. It also gave rise to his collaboration on Peano's *Formulaire des Mathématiques*, as shown by documents in the Peano-Vacca Archive, now conserved in the Dep. of Mathematics at the University of Turin. The five volumes of the *Formulaire* with *marginalia* by Vacca and Peano, the thousands of pages of Vacca's manuscript notes, as well as his lecture notes for his courses in Logic and History of Mathematics in Genoa and Rome, all document – even more amply than his publications do – his activities in this field and his working method, refined during his long years of militancy in Peano's 'School'.

When Vacca first came into contact with Peano, in 1894, the history of logic had already been sketched several times in Peano's technical writings, which often opened with a brief historical overview. According to Peano, the father of logic was Aristotle, who was followed by G.W. Leibniz, the first to have occupied himself with it to a large extent, formulating analogies between operations of algebra and those of logic. Only in the nineteenth century, with the German and English logicians such as G. Boole, A. De Morgan, E. Schröder and others, was a systematic treatment arrived at.[2] To be sure, this was a bare bones account, but it was later fleshed out, thanks to the investigations of Peano himself. For example, to Peano we owe the rediscovery in 1894 of the attempts to construct a *characteristica universalis*, along the lines of a project of Leibniz, carried out by the Piedmontese L. Richeri in his *Algebrae philosophicae in usum artis inveniendi, specimen primum*, published in 1761 in the *Miscellanea* of the Turin Academy of Sciences.[3]

Research in this area intensified with the publication of the *Formulaire*, an ambitious encyclopedia of elementary mathematics, prepared in logical-ideographic form, in which every notation, definition and theorem had to be

ria), 2, 1899, p. 113-116; "Sui precursori della logica matematica", *RdM*, 6, 1899, p. 121-125, 183-186; "Additions au Formulaire", *RdM*, 7, 1901, p. 59-66; "La logica di Leibniz", *RdM*, 8, 1903, p. 64-74; "Maurolycus, the first discoverer of the principle of mathematical induction", *AMS Bull*., 2, 16, 1910, p. 70-73; "Sulla storia del principio d'induzione completa", *Boll. Bibl. Sto. Sci. Mat. (Loria)*, 12, 1910, p. 33-35; *Rev. Méth. et Morale*, 19, 1911, p. 30-33; *Origini della scienza*, Roma: Partenia, 1946. Cf. Luciano, E., Roero, C.S. (2010): *Giovanni Vacca* in Roero, C.S. (ed.): *Peano e la sua Scuola fra matematica, logica e interlingua. Atti del Congresso Internazionale di Studi* (Turin 6-7.10.2008), Turin: DSSP, p. 98-113.

[2] Peano, G. (1891): "Principi di logica matematica", *Rivista di Matematica* (*RdM*), 1, p. 1.

[3] Peano, G. (1894): "Un precursore della logica matematica", *RdM*, 4, p. 120.

accompanied by information of a historical-bibliographical nature. The first edition dated to 1895, and the historical part was prepared by Peano and Vailati. However, Peano himself understood that it was full of gaps, especially the part regarding logic, whose propositions were attributed, sometimes erroneously, to Leibniz, Boole, Peirce, Aristotle, MacColl, Segner, De Morgan, Schröder, Hauber, Jevons and Dedekind. He thus decided to turn to Vacca, whose article on a proof by Leibniz of the theorem of Fermat-Wilson published in Eneström's journal *Bibliotheca Mathematica* he had much admired. Citing Vacca's research 'with which you are occupied, with such good results', Peano invited the young mathematician, who at that time had just received his degree, to assist him in compiling historical notes for the *Formulaire*.[4] The offer was immediately accepted,[5] and thus the *Formulaire* was substantially enriched, so much so that from the historiographical point of view, the essential difference between the first and successive editions consists precisely in the thousands of additions and corrections both historical and bibliographical made by Vacca.

The heterogeneous nature of the arguments and authors that Vacca found himself examining in order to prepare the notes for the *Formulaire* might lead one to believe that Vacca's research activity was of an episodic and almost *naïf* nature, but this was not the case. The guidelines for preparing the historical-bibliographical apparatus, dictated by Peano, were in fact quite strict and corresponded to a very precise historical approach, outlined in 1898. The notes had to go back to the origins of the passages cited in order to facilitate comparison and a direct reading; they were to be as precise as possible, and to provide the transcription of extracts of significant works and correspondence, in order to make evident 'l'importanza delle proposizioni, e spesso il vantaggio dell'ideografia'.[6] Viewed this way, the history of mathematics thus becomes a 'history of mathematical rigour' and is conceived in strict connection to a particular trend in the research on mathematical logic and the desire to create a 'literary criticism' of the classic texts of mathematics. The dimension of history, which Vacca claims to be as important for active researchers as well as historians, makes it possible to 'found' mathematics, in the sense that it makes evident the steps to its formalization and the evolution of the theories towards the structure of hypothetical-deductive systems. This kind of approach nevertheless poses objective limits, such as the fact that it lead to diffi-

[4] G. Peano to G. Vacca, 15 May 1894, c. 1r, *Peano-Vacca Archive*, Turin.
[5] G. Vacca to G. Peano, 31 May 1894, c. 1r-v, *Peano-Vacca Archive*, Turin.
[6] Peano, G. (1898): "Sul §2 del Formulario, t. II: Aritmetica", *RdM*, 6, p. 83.

cult investigations into priority disputes and the research for forerunners, according to a canon of interpretation that was then at its height, although today it has fallen into disuse.

The first objective Vacca set for himself in order to 'write the history' in the *Formulaire* was aimed at filling a gap in Peano's reconstruction that appeared unnatural to him: the one between Aristotle's logic and the modern developments due to Leibniz. Taking as his point of departure a work by Loria, *La logique mathématique avant Leibniz*, Vacca began to study the work of authors both famous and obscure, in which he traces results that were noteworthy both in terms of symbolism, and in terms of the logical analysis of the ideas of the exact sciences. The method used to represent the proofs and the beginnings of pasigraphy found in P. Hérigone's *Cursus Mathematicus* (1644); the symbolism for the operators and the symbol of *illatio* (inference) employed by J. Pell in his *Introductio in Algebram* (1668); the systemisation of the ideas of geometry expounded by L. Carnot in his *De la corrélation des figure de géométrie* (1801); and finally the use of symbols to indicate relations and the problem of the 'definition of new words' in the *Essai de dialectique rationnelle* (1816) and in the *Essai sur la théorie des définitions* (1818) by J. Gergonne are only some of the topics developed by Vacca in the article entitled *Sui precursori della logica matematica* (1899), which became a classic.

2. The rediscovery of Leibnizian manuscripts

The slender nature of these contributions strengthened Vacca's conviction about the diversity of roles played by Leibniz in the history of logic, from both a qualitative and a quantitative point of view. 'Leibniz the logician' was only slightly known at the time, since a minimum of his manuscripts had been included in the collections of his papers edited by L. Dutens (1778) and by C.I. Gerhardt (1849, 1875). In light of what he had learned from the study of Leibniz's *Opera philosophica* edited by J.E. Erdmann (1840), and the catalogue of Leibniz's correspondence compiled by E. Bodemann (1889), in July 1899 Vacca travelled to Hannover, suspecting some 'lacune nei lavori stampati'.[7] As can be seen from ten or so pages of handwritten notes, he was able to consult manuscripts of Leibniz which were catalogued under the signature *Mathematik* 3B XI, fol. 10; 3A 3, fol. 16 e *Philosophie* 7B 4, fol. 1-3, 17 e 7B

[7] Peano, G. (1904): "Il latino, quale lingua ausiliare internazionale", *Atti R. Acc. Sci. Torino*, 39, p. 277-278.

2, fol. 3, 17. The first two of these made it possible for Vacca to attribute to Leibniz some important results in number theory and binary arithmetic. The next two led him to dispel the inexact conclusions regarding Leibniz's contributions to mathematical logic. From Hannover Vacca sent Peano the transcripts of several passages of these *brouillons*. These were written in the margins of the print galleys of the *Formulaire* (1898-99) and inserted into the additions and corrections that would be published in Peano's *Rivista di Matematica*, and would later be integrated into successive editions of the work.

Due to its structure and aims, the *Formulaire* only gives the transcriptions of some excerpts of Leibniz's manuscripts. While philologically accurate, these are not integrated into a systematic historiographical treatment; they are neither accompanied by interpretations or comments, nor are they intended as a prelude to a later publication specific to history of logic. For his part, Vacca was always reluctant to produce copious monographs; moreover, during his five years of collaboration on the *Formulaire*, he had honed his particular vision of research, in which 'doing the history of a science' meant more particularly "chercher et exposer dans le passé tous les essais qui ont produit successivement les vérités que nous connaissons. […] L'histoire d'une science est alors l'exposition ordonnée des vérités de cette science suivie d'un nome ou d'un date".[8] It isn't therefore surprising that he limited himself to merely making mention in passing of his investigations in a brief paper in the *Bollettino di Bibliografia e Storia delle Scienze Matematiche* of his friend Loria, referring the reader to the *Formulaire* for 'a precise examination' of Leibniz's manuscripts.[9]

Just as it is possible to identify clearly the methodological assumptions of Vacca's approach to the history, it is also possible to identify what he effectively 'discovered' regarding Leibniz's logic. We need only compare the first two editions of the *Formulaire* to ascertain that, following his stay in Hannover, 28 of the 30 propositions of the entry for logic are attributed to Leibniz. What especially inspired Vacca's admiration of the philosopher-mathematician's 'genius' was his knowledge of the properties of the sign for negation, the identity of the symbol for deduction used among classes and propositions, some relations among the logical symbols and the propositions regarding the divisibility of whole numbers, and finally, the 'so evocative and

[8] G. Vacca to L. Couturat, 1901, in Nastasi, P., Sciamone, A. (eds.) (1995): *Lettere a Giovanni Vacca*, Palermo: Quad. Pristem, 5, p. 51.

[9] Vacca, G. (1899): "Sui manoscritti inediti di Leibniz", *Boll. Bibl. Sto. Sci. Mat. (Loria)*, 2, p. 115.

elegant representation of the forms of reasoning by means of systems of circles, ordinarily attributed to Euler'.[10]

The examination of Leibniz's unpublished manuscripts marked the beginning of Vacca's collaboration with Louis Couturat. The two met in Paris in 1900, on the occasion of the International Congress of Philosophy, and Vacca told the French philosopher about the immense quantity of unpublished papers that lay 'sepolti' (entombed) in the library in Hannover. At the time Couturat was just about to publish *La Logique de Leibniz* (Paris, 1901), the first of his books on Leibniz. Carefully following Vacca's indications, he modified some parts of the work, and for this reason it was enthusiastically received by members of the Peano School, as the reviews by Vacca and Vailati in the *Rivista di Matematica* show.

Intent on pursuing the research begun by Vacca, in spring 1901 Couturat travelled to Hannover, where he stayed for a year. From this moment the preparation of *Opuscules et fragmentes inédits de Leibniz* (Paris, 1903), his second important work on Leibniz, was the subject of on-going discussions between Couturat, Peano, Vacca and Vailati. However, it was above all with Vacca that Couturat discussed the choice of manuscripts best suited to showing the advantages of algorithmic logic and, following his advice, he decided to include in his collection several passages from Leibniz's correspondence with the Jesuits in China.[11] At the same time, Vacca and Vailati, who were sent copies of the print galleys of the *Opuscules*, helped Couturat to correct the galleys, also discussing some interpretations that were only slightly documented. Couturat's appreciation of Vaccca's input is found in the preface to the *Opuscules*. For his part, asked to review the book for the *Rivista di Matematica*, Vacca took the opportunity to complete the outline for his own more systematic publication on history of logic. Taking as a point of departure J.B. Say's criteria, according to which: "l'histoire d'une science ne ressemble point à une narration d'événements. Elle ne peut être que l'exposé des tentatives plus ou moins heureuses, qu'on fait à diverses reprises et dans plusieurs endroits différents, pour recueillir et solidement établir les vérités dons elle se compose. … Les erreurs ne sont pas ce qu'il s'agit d'apprendre, mais ce qu'il

[10] *Ibid*., p. 115. It should be noted that representation by means of Euler-Venn diagrams was also used in the classes in mathematical logic taught by Vacca in Genoa (1903) and by Peano in Turin (1906). Cf. Peano, G. (1900): "Formules de logique mathématique", *RdM*, 7, p. 9 and Vacca, G. (1903): *Elementi di Logica Matematica, Estratto dalle Letture fatte nella Università di Genova nel 1903*, Genoa, p. 1-24.

[11] Cf. Luciano, E. (2012): p. 48-55.

faudrait oublier"[12], Vacca once again takes the stance from the point of view of a mathematical logician with a Peano-inspired outlook more than that of a professional historian. Although without neglecting to frame Leibniz in relation to his precursors and heirs (especially Gergonne and Lambert), he proposes a comparison between the unpublished manuscripts of the *Opuscules* and the presentation of logic drawn from the *Formulaire,* going to 'seek in the manuscripts of Leibniz the parts which are today most interesting for mathematical logic in its present state'.

3. Vacca's contributions after 1903

The rediscovery of the unpublished papers of Leibniz helped draw the attention of other historians of science to logic, including that of G. Itelson, who at the International Congress for Historical Sciences held in Rome in April 1903, hinted at the contributions of J. Jung, J.-C. Sturm, Moulin and Langeius. It would be precisely beginning with this presentation that Vacca and Vailati would intensify their efforts, in the early years of the 1900s, to locate manuscripts on logic and authentic bibliographic rarities in European libraries.

From 1903 on – paradoxically, if we think of the successes achieved by logic in those years and the fact that it had started to become a discipline in its own right – Vacca's interest in the history of logic diminished. He became convinced of the fact that, for this area, 'the *era of discoveries* is at an end, or close to the end, for a long time to come'.[13] At the same time he began to nourish a discrete and finitist vision of historical processes that were constellated by the presence 'at long intervals of time and space, [of] some small groups of men, hardly numerous enough to be able to talk to each other, to whom alone it is given to create beautiful and noble inventions'.[14]

So it came about that, with the exception of some traces of interest in the Scholastics Pedro Hispano, Buridan and Llull, and then some works on the principle of induction (1910-11), in which Vacca traces its use – well before

[12] Say, J.-B. (1840): *Histoire abrégée de l'économie politique. Cours complet …*, Paris: Guillaumin, Part II, p. 540.

[13] [*chiusa o prossima a chiudersi l'*era delle scoperte *per un lungo intervallo di tempo*]. G. Vacca to G. Vailati, 7 November 1903, in Lanaro, G. (ed.) (1971): *Giovanni Vailati. Epistolario 1891-1909*, Torino: Einaudi, p. 226-227.

[14] [*a lunghi intervalli di tempo e di spazio, [di] alcuni piccoli gruppi di uomini, appena tanti quanti occorrono per poter parlar l'un coll'altro, ai quali soli è dato creare belle e nobili invenzioni*]. G. Vacca to M. Pieri, 12 September 1912, cc. 1-9, *Peano-Vacca Archive*, Turin.

B. Pascal – back to Euclid, Nicomachus of Gerasa, Campanus of Novara and Maurolico, Vacca's contributions to the historiography of logic became fewer and less profound. For the most part dedicated to the contemporary phase, and sometimes marred by ingenuous and excessively pompous tones, they present Peano as the point of arrival for the entire evolutionary course of logic, and flatten the contributions of other authors, who are indicated indiscriminately as precursors of his *Maestro*.[15] Having played a leading role, or at least the second lead, in the recent developments of logic, Vacca inevitably had a hard time achieving the necessary detachment with regard to these topics, and evaluating lines of research other than those of Peano.

Vacca's work shared the limit of contemporaneousness with another monograph that appeared in those same years: F. Enriques' *Per la Storia della Logica* (1922), which was immediately disseminated internationally thanks to translations into French, German and English (1925, 1927, 1929). A colleague of Vacca at the University of Rome, Enriques's relationship with Vacca was friendly, and his esteem for Vacca's research was profound, so much so that he entrusted him with the course in History of mathematics at the Istituto di Storia della Scienza that he had founded. A reader with little regard for sources, Enriques frequently thanked Vacca for numerous bibliographical references that he had provided for the texts and manuscripts by Gergonne, Buridan and Paul of Venice.

Vacca and Enriques nevertheless found themselves on two sides of a deep divide concerning the concept of logic – and thus of its history. To use an image dear to Enriques:

> "… there is, if you will, a small-scale logic and a large-scale logic: I mean the refined analysis of the process of exact thought (almost a kind of microscopic view of the elements that make up the fabric of science), and – to the contrary – the study of the systematic connections of the system, that is, the macroscopic view of science".[16]

Enriques was thus concerned with the history of 'large-scale logic': the history of its connections with the philosophy, gnoseology, epistemology and psy-

[15] Cf. Vacca, G. [1939]: *La Logica matematica negli ultimi cinquant'anni*, ms., p. 1-13, *Peano-Vacca Archive*, Turin.

[16] [*vi è, se cosi è lecito esprimersi, una logica in piccolo ed una logica in grande: intendo l'analisi raffinata del processo del pensiero esatto (quasi la veduta microscopica degli elementi che formano il tessuto della scienza), e - per contro - lo studio delle connessioni organiche del sistema, cioè la veduta macroscopica della scienza*]. Enriques, F. (1921): "Insegnamento dinamico", *Periodico di Mat*., (4) 1, p. 6.

chology of mathematics, while Vacca was essentially concerned with a history of 'small-scale logic', and more precisely, a history of ideographical logic. Enriques's approach is attentive to the becoming and continuity, and is aimed at providing dynamic interpretations in a rationalist philosophical perspective. He arrives at the construction of evocative choral syntheses, even if his vision of the whole is not always joined to perfect accuracy of philological analysis and meticulousness of reconstructions. In contrast, Vacca's historical methodology was strictly concentrated on the sources, more prone to appreciate the fragment, the minor author, the subtle contribution, even at the risk of crossing the line between history and erudition.

The hermeneutic attitude and the gusto for raising questions and hazarding historiographical hypotheses, and, on the other hand, the philological rigour and the passion for the unpublished, are always constitutive traits, and frequently happily combined, in historians of science. At a time when Italy was on the cutting edge internationally in research in mathematical logic, the figures of Vacca and Enriques mark a significant moment in the professionalization of historiographical research, anticipating with their contributions a tradition of studies which would become consolidated starting in the 1920s.

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