

A World in Time

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ABSTRACT. This paper is intended to analyse the concept of time in the Gödelian models of Einstein's field equations, by showing the intimate connections between Gödel's views on (the philosophy of) logic and mathematics and his cosmological work. He came up with these models, when he was working on a paper for a book of the *Library of Living Philosophers* devoted to Einstein about the concept of time in theory of relativity and in the Kantian philosophy. Gödel provided the mathematical structure of some Rotating worlds which are coherent with the theory of relativity, but within which an objective dimension of time cannot be constructed (see Section 1). In Section 2, I underline the role of the Kantian philosophy on time and the particular interpretation that Gödel has offered, which has led him to become a holder of the idealistic view on time. Section 3 will be devoted to Gödel's argument for the ideality of time, while in Section 4 I will make the role of the concept of "possible world" in Gödel's works clear as well as his importance for his views on time. Finally, in Section 5 I will present my concluding remarks which show the link between the Kantian dimension of time in modern physics and the structure of our cognition.

KEYWORDS: time, Gödel, Rotating worlds, possible worlds.

Ogni cosa mortal Tempo interrompe.

F. PETRARCA, *Trionfi*

1. Rotating worlds

The concept of time is a perennial issue in philosophy and in the sciences. It is quite standard to follow McTaggart's distinction between the A-series of time, in which events can be ordered by way of the non-relational predicates "is past", "is present" and "is future", and the B-series in which events can be ordered according to the relational predicates "comes before" and "comes after". Moreover, it is also important to make a distinction between an idealistic view on time and a realistic one, in order to have a better framework on the problem of time. Idealists maintain that time and change are not real, because time is Plato's "moving image of eternity", while realists maintain that time is an essential relation of the events which occur in our world.

After the formulation of the theory of relativity, many philosophers have argued that the philosophical problem of time still remains. An exception to this claim has been provided by Putnam (1967): "I do not believe that there are any longer any philosophical problems about Time; there is only the physical problem of determining the exact physical geometry of the four dimensional continuum we inhabit". On the contrary, Gödel argues: "Time remains the philosophical problem even after Einstein" (Wang 1996); and: "Time is that mysterious and seemingly self-contradictory being which, on the other hand, seems to form the basis of the world's and our own existence" (Gödel 1949b).¹ Gödel acknowledges that time is a fundamental issue in philosophy and that the question of time cannot only be placed upon the framework of the theory of relativity. The concept of time in the theory of relativity can be well understood if we read the following quote from Heisenberg:

In the theory of relativity future and past are separated by a finite time interval the length of which depends on the distance from the observer. Any action can only be propagated by a velocity smaller than or equal to the velocity of light. Therefore, an observer can at a given instant neither know of nor influence any event at a distant point which takes place between two characteristic times. The one time is the instant at which a light signal has to be given from the point of the event in order to reach the ob-

¹ Yourgrau (2005) is an important monograph dedicated to Gödel and Einstein and their works in the theory of relativity. The previous quotes can be found in Yourgrau's book too.

server at the instant of observation. The other time is the instant at which a light signal, given by the observer at the instant of the observation, reaches the point of the event. The whole finite time interval between these two instants may be said to belong to the “present time” for the observer at the instant of observation. Any event taking place between the two characteristic times may be called “simultaneous” with the act of observation (1958, p. 71).

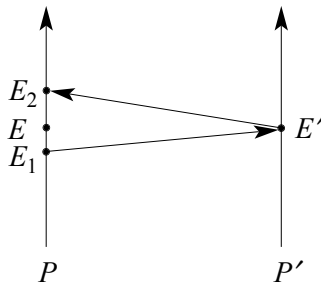
Thus, the “concept of time” needs the “concept of simultaneity” if we want to understand the difference between past and future and to grasp the relativistic present, otherwise we cannot apply any time-metrics, e.g., we need simultaneity between an event and a clock. Furthermore, our conception of time requires inertia, since we need to place the events in time according to the Kantian “enduring magnitude”, which can be considered as a sort of physical inertia. The concept of inertia plays a very important role in the development of the theory of relativity. In effect, Gödel has proposed a solution of Einstein’s field equations in the theory of relativity, which allows for Rotating worlds, i.e., worlds where the “compass of inertia” rotates everywhere relative to matter, namely it rotates relative to the totality of the galactic systems with angular velocity $\sqrt{2(\pi\kappa\rho)}$, where ρ is the density of matter and κ is Newton’s gravitational constant.

A Rotating world (R-world) presents an inertial field, which can be considered as a possible “substitute” for the “absolute space”, which determines the motion of bodies upon which no force acts. The inertial field determines the behaviour of the axis of the pendulum at the pole, where no force acts on it. Matter rotates with respect to this inertial field of the pendulum. Hence, the R-world does not rotate itself as a whole with respect to a particular axis, but this is a different kind of inertia which allows the rotation of the world. Nevertheless, the mutual distances of any two material particles remain constant all of the time. R-worlds present Timelike Closed Lines, which are not geodesics (namely, the shortest trajectories between two points in a particular space). “Timelike” means that the space-time lines do not exceed the speed of the light, while “close” means that the lines create a loop, which allows for time travels. Notice that it is possible to get only a partial ordering between events, because if one tries to define an absolute “before”, then the events cannot be compared or they lay in a cyclic ordering. Moreover, there is no objective lapse of time and there is no linear ordering because the temporal distances lack the property of additivity in a metric space. Namely, what Gödel wants to show is that our intuitive concept of “time” cannot be elucidated in an R-world.²

² See Yourgrau (2005).

He observes that if two events are simultaneous for one observer, it can be the case that they are not simultaneous for another observer, if he is in motion relative to the first observer. Hence, Gödel remarks that the statement: “‘A before B by t seconds’ is certainly something not inherent in the events, nor does it allow time to be a substratum in which the events are lying” (Gödel 1946/49, p. 250).

Notice that in the R-worlds, differently from the theory of relativity, it is not possible that a first signal (as presented in Reichenbach [1928] 1958, Section 22) can occur. To clarify the concept of *first-signal*, imagine the following situation. We want to send a signal from P to P' . We call E_1 the event of the departure of the signal from P , while E' is the event of its arrival on P' . Simultaneously at the arrival of the signal, another signal is sent from P' to P . Let the event of the arrival of the signal from P' to P be called E_2 . Therefore, the order of E' relative to E_1 and E_2 is the following: $E' > E_1$ and $E' < E_2$. Consider now an event E on P for which $E_1 < E < E_2$. We could ask which is the position of E relative to E' . If the first signal is sent from P then $E > E'$, while if the first signal is sent from P' then $E' > E$. Thus, E and E' cannot be correlated, namely they are indeterminate in respect to the time order. Of course, this requires the velocity of light to be limited, since if one assumes the Newtonian framework within which the velocity of light is infinite, then the interval $E_1 - E_2$ will be reduced to a point E_0 . In addition, if we accept that the velocity of the signal is finite, then the first signal determines a finite interval and not a pointlike event, e.g., we can provide a topological definition such that “any two events which are indeterminate as to their time order may be called simultaneous” (*ibid.*, p. 145) and therefore any event which occurs in P between E_1 and E_2 is indeterminate and simultaneous with E' .



In case of an R-world, we cannot have a first signal, since from every space-time point a signal can be sent to every other point, which goes back to its de-

parture point, since the trajectory is a circle. This implies that every observer has his own “now”, which cannot represent the objective flow of time. A similar point on the role that the concept of “now” plays was addressed by Einstein in some conversations with Rudolf Carnap, which have been reported in his intellectual autobiography.³

Gödel observes that another important feature is that his solution of Einstein’s field equations seems to be not coherent with *Mach’s principle*, which is, in any case, a principle logically independent of the theory of relativity. Mach’s principle greatly inspired Einstein in the development of the theory of relativity. This principle states that: “mass there influences inertia here”, i.e., position and velocity are relative concepts and this fact implies that even our concept of inertia needs to be relativized.⁴ Mach presented this principle in his *Mechanics* (but the principle is nowadays known after that Einstein popularized it in 1918) where he attacked the Newtonian idea of the absolute space and the absolute time, because he argues that all the principles of mechanics are based on our experience of relative locations and relative motions, hence also the concept of inertia should be revisited.⁵ Thus, according to Mach, in-

³ “Once Einstein said that the problem of the Now worried him seriously. He explained that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics. That this experience cannot be grasped by science seemed to him a matter of painful but inevitable resignation. I remarked that all that occurs objectively can be described in science; on the one hand the temporal sequence of events is described in physics; and, on the other hand, the peculiarities of man’s experiences with respect to time, including his different attitude towards past, present, and future, can be described and (in principle) explained in psychology. But Einstein thought that these scientific descriptions cannot possibly satisfy our human needs; that there is something essential about the Now which is just outside the realm of science. We both agreed that this was not a question of a defect for which science could be blamed, as Bergson thought. I did not wish to press the point, because I wanted primarily to understand his personal attitude to the problem rather than to clarify the theoretical situation. But I definitely had the impression that Einstein’s thinking on this point involved a lack of distinction between experience and knowledge. Since science in principle can say all that can be said, there is no unanswerable question left. But though there is no theoretical question left, there is still the common human emotional experience, which is sometimes disturbing for special psychological reasons” (Carnap in Schilpp 1963, pp. 37-38).

⁴ In fact, in general relativity, the *principle of equivalence* – between the “freely falling trajectories in a *gravitational field* and the trajectories described by the free particles affected by no force at all – replaces the classical law of inertia holding in both Newtonian mechanics and special relativity” (Friedman 2008, p. 108).

⁵ Isaac Newton presented his well-known “rotating bucket argument” in order to show the difference between an absolute motion and a relative one and between the absolute space and time and their relative counterparts. Mach would interpret that experiment without any reference

ertia is not an internal property of a single body anymore, but rather the consequence of the influence of all the other bodies of the universe. But in the R-worlds the inertial field is independent of the state of motion of matter, hence Mach's principle does not hold in such kind of world (see Gödel 1949c, p. 271).

Another feature of the R-worlds is that we could travel *in principle* in every region of the past, the present and the future with a space-shuttle, but actually we cannot travel in time since the fuel needed by the shuttle would be too many times greater than the weight of the shuttle. Nevertheless, Gödel argues that the impossibility which occurs in practice can be discovered later as a kind of impossibility which occurs also *in principle*⁶ as in the case of the Uncertainty principle.

2. Gödel, Kant and the concept of time

Gödel maintains that it is possible to assign to the concept of time both a negative and a positive definition. According to the negative definition, time is not a characteristic or ordering inherent in the objects, while according to the positive definition, time is a characteristic concerning the relation of the objects with something else and time is the a priori condition of possibility of our sensibility. Furthermore, Gödel maintains that our intuitive conception of time implies that time is a system of one-dimensional points, isomorphic with a straight line and that time flows.

Gödel considers the theory of relativity as a step further with respect to the Kantian philosophy towards a more objective grasp of the concept of time. Nevertheless, he argues that the idealistic account of time cannot be falsified by the theory of relativity itself, because of the difficulties to compare time in physics with time in our everyday experience. He writes:

Change becomes possible only through the lapse of time. The existence of an objective lapse of time, however, means (or, at least, is equivalent to the fact) that reality consists of an infinity of layers of "now", which comes into existence successively. But, if simultaneity is something relative in the sense just explained, reality cannot be split up into such lay-

towards absolute concepts and would modify Newton's concept of inertia. For the history of the concept of Mach's principle, see Lichtenegger and Mashhoon (2007).

⁶ On the distinction between "effective" and "in principle", see Pataut (2002) which is an interview to Dummett.

ers in an objectively determined way. Each observer has his own set of “nows”, and none of these various systems of layers can claim the prerogative of representing the objective lapse of time (Gödel 1949b, pp. 202-203).

It seems that Gödel considers the hypothesis for which the concept of “present” can be not a point-like dimension of the objective time, but a subjective dimension of time with duration which splits up the events in time relative to an observer. This latter view resembles the construction of the intuitionistic continuum and the *ur-intuition* (or *two-ity*) of mathematics.⁷ But Gödel claims that such subjective (intuitionistic) dimension of time cannot stand for any temporal objective level, because it is relative to the position of the observer and because the concept of existence cannot be relativized to a particular observer in a particular context.⁸ Using a modern terminology, we can say that Gödel maintains an endurantist view in ontology, since objects exist without any reference to temporal parts and temporal dimensions.⁹ I will come back on the issue of Gödel’s account of the concept of existence in Section 4, because

⁷ “[...] the ur-intuition of mathematics (and of every intellectual activity) is the substratum, divested of all quality, of any perception of change, a unity of continuity and discreteness, a possibility of thinking together several entities, connected by a ‘between’, which is never exhausted by the insertion of new entities. Since continuity and discreteness occur as inseparable complements, both having equal rights and being equally clear, it is impossible to avoid one of them as a primitive entity, trying to construe it from the other one, the latter being put forward as self-sufficient; in fact it is impossible to consider it as self-sufficient. Having recognized that the intuition of continuity, of ‘fluidity’, is as primitive as that of several things conceived as forming together a unit, the latter being at the basis of every mathematical construction, we are able to state properties of the continuum as a ‘matrix of points to be thought of as a whole’” (Brouwer 1907, p. 17).

⁸ On the constitutive dimension of the intuitionistic intervals of time (which have a duration and need an intuitionistic *continuum*) as opposed to the realistic views on time, see Dummett (2000).

⁹ Recently, some analytical philosophers have started being interested in Gödel’s R-worlds, because of their importance in ontology. Ted Sider (2001) maintains that endurantism cannot hold in an R-world. He endorses Lewis’ view for which temporary intrinsic properties (such as “being charged”) exist as dyadic relations between objects and times. But in the R-world it is impossible to define time, therefore endurantism does not hold, because there is no dyadic relation with time. Effingham and Melia (2007) have replied to Sider that the endurantist position can hold in the R-worlds if we analyse the property of “being charged” as a relation between individuals and points of a manifold, which can stand for times, or temporal parts or just points in a structure without any reference to times when, for instance, they are interpreted in the R-worlds. In any case, Gödel’s account of the concept of “existence” of an object does not require any dyadic temporal relation, because it is an absolute concept.

firstly I want to analyse the Kantian features of time, which turn out to be important for the Gödelian remarks on the concept of time.

According to Kant, we cannot experience “Time itself” (B.219) and we cannot place the objects in this absolute time. Hence, we need an indirect way to determine the temporal ordering of the objects through the distinction between a subjective and an objective dimension of time which is very important in Kant’s philosophy, i.e., the cognitive dimension of time does not coincide with the objective account of it.¹⁰ This distinction is based on the further dichotomy between the constitutive figures of experience and the regulative ones (which rule a pre-existing state of affairs). Consider the structure of the *Critique of Pure Reason*. In the Analytic of Principles we apply every particular category to the sensible intuition, while this function was done in the transcendental deduction a priori and in a more general way. In this way, Kant makes a distinction between the Axioms of Intuition and the Anticipations of Perception, which are constitutive features of the experience and the Analogies and the Postulates of the Empirical Thought, which are regulative. The second and the third analogy of experience present the dimension of causality and the connections among objects through their mutual interaction. By contrast, in the Transcendental Aesthetic, the constitutive element of space and time is almost merely subjective and it is an *empty* form of sensibility, while with the analogies we achieve a more objective framework about the nature of the objects, where space and time are conceived in their physical fullness.¹¹ Moreover, Kant refers to the *modi of time* (namely Persistence – time as an enduring magnitude –, Succession and Simultaneity), because he wants to present a unitary framework for time, which can allow for an adequate interplay between the subjective (and constitutive) dimension of time and the objective (and regulative) dimension of it. Persistence refers to the concept of time intended as a magnitude or a force, while Succession and Simultaneity alone cannot achieve to present the “unity of time” without a persisting time.¹²

¹⁰ See the example of the ship and the house in the *Critique of Pure Reason*, B 232.

¹¹ See Watkins (2005). The Kantian distinction between the *regulative* and the *constitutive* features of knowledge was taken into consideration by many philosophers. See Quine (1948), where this distinction is presented making a comparison between the Medieval dispute over universals and the philosophy of mathematics of the first half of the last century. Quine noticed that the constitutive aspects of our knowledge correspond to an intuitionistic account of mathematics, while classical mathematics deals with the entities which pre-exist to any definition, namely a regulative role. This distinction plays also an important role in philosophy of language. See Searle (1969, pp. 33-42).

¹² Notice that there are some formal counterparts of these Kantian *modi of time*, e.g. Succession and Simultaneity are almost equivalent to the relations *P* (precedes) and *O* (overlaps) in

Gödel's remarks on Kant's conception of time do not take into account the dimension of the objective time. In Gödel (1946/49) there is no discussion about the Analogies of the Experience, because he bases his Kantian remarks especially on the *Prolegomena* and the Transcendental Aesthetic. As a matter of fact, the Analogies of Experience are concerned with the existence of appearances, the Transcendental Aesthetic with the mere form of their sensible intuition (see Longuenesse 1998, chapter 11). Therefore, he interprets Kant's concept of time in a subjectivist way, because it is in the Analogies of Experience that time has a more objective dimension. Nevertheless, some Gödelian remarks on the Kantian structure of cognition could result interesting, even when they are compared with the theory of relativity and the non-Euclidean geometry. Consider the following quote:

The fact that the physical bodies surrounding us move by the laws of a non-Euclidean geometry does not exclude in the least that we should have a Euclidean "form of sense perception", i.e. that we should possess an a priori representation of Euclidean space and be able to form images of outer objects only by projecting our sensations on this representation of space, so that, even if we were born in some strongly non-Euclidean world, we would nevertheless invariably imagine space to be Euclidean, but material objects to change their size and shape in a certain regular manner when they move with respect to us or we with respect to them (1946/49, p. 255).

Gödel wants to show that our Euclidean intuition would be a priori valid even if the structure of the world were non-Euclidean. Gödel states that our Euclidean intuition is innate and a priori valid even in a non-Euclidean world. Actually, Klein proposed in 1871 a simple projective model of hyperbolic geometry, in which the fundamental entities are the same entities of the Euclidean geometry and in which the points of the geometry are in an n -dimensional disk, or ball, and the lines of the geometry are line segments contained in the disk. In this way it is possible to map non-Euclidean objects with Euclidean ones.¹³

Gödel argues that the temporal relations represent relations between the perceiving subject and the objects. Namely, *Schein* stands for the appearance of the things in themselves. *Erscheinung* stands for the appearance of the

the Russell-Walker calculus (see Russell 1914, 1936; Walker 1947), while Persistence is equivalent with the concept of fluent in the event-calculus (see van Lambalgen and Hamm 2004).

¹³ A similar idea was envisaged in 1884 by Edwin Abbott in his novel *Flatland: A Romance of Many Dimensions*.

things with the sensibility of the subject. Appearances are directed towards objects of thought which have the properties of the objects of our sensibility and they are neither mere illusions nor properties of the things in themselves, but their possibility is justified by the existence of the things in themselves. Having this framework in mind, we can analyse Gödel's argument of the a priori validity for the Euclidean intuition. The apriority of space and time refers to the objective counterpart of these ideas but, through the *Erscheinung*, the appearances of the objective space and time are connected with the innate schema of sensibility of the subject. Therefore, even the subjective space and time are respectively applicable in the forms of Newtonian time and Euclidean space to the physical reality (as a kind of relative space and time) as we saw in the case of Klein's geometrical translation. Notice that the arguments about the apriority of space and time hold if one maintains that there is an innate Euclidean intuition, which is merely assumed by Gödel. When Gödel refers to the Kantian objectivity of time and space, he does not take into account the penetrating analysis of these concepts in the *Transcendental Analytic* (as I have stressed above). Gödel's epistemic views support the idea that we can construct our knowledge with different levels of objectivity of the phenomena that can lead us towards the very essence of the things in themselves (noumena). Hence, the Kantian world of appearances (with "many subjective elements of the world of the sensations" according to Gödel) is considered to be the first step of the objectivation of reality if it is compared with the theory of relativity, which is a further step in this process with the aim to go beyond the appearances and approach the things in themselves partially and step by step.

3. Ideality of time

Gödel presented an argument to show the idealistic features of time in his R-worlds and in our actual world. Apparently, such claim does not seem to fit with Gödel's Platonistic views, but a further analysis can explain this apparent contrast between his ontology and his account of the concept of time. Gödel's argument shows the ideality of time in the R-worlds and from that he infers the ideality of time in our world.¹⁴ The complete argument runs in the following way:

¹⁴ Gödel partially assumes a Leibnizian point of view on the nature of the individuals and the physical laws. In fact, Leibniz believed that an individual is an inhabitant of one single world, and every other world with different physical laws would determine a different individ-

- (0) Time is real only if change is real;
- (1) Change is real only if there exists an objective lapse of time;
- (2) Time is real only if there exists an objective lapse of time [from (0) and (1)];
- (3) “The existence of an objective lapse of time means or at least is equivalent to the fact, that reality consists of an infinity of layers of ‘now’ which come into existence successively” (Gödel 1949b, p. 202);
- (4) Reality consists of an infinity of layers of “now” which come into existence successively only if space-time admits of a global time function (*cosmic time*);
- (5) Time is real only if space-time admits of a global time function [from (2), (3) (4)];
- (6) Gödel’s rotating-model M is a solution to Einstein’s field equations, hence it is a physically *possible* model;
- (7) Since for every x in M , x chronologically precedes itself, M does not possess a global time function;
- (8) In the physically possible world M , time is ideal [from (5) (6) (7)];
- (9) The main, contingent, non-lawlike difference between M and our universe is given by the (probable) absence of a net rotation of matter, which implies the existence of cosmic time in our world;
- (10) ?;

therefore, time is ideal also in our universe (this rigorous presentation of the argument is due to Dorato 2001).

Gödel assumes that change and time are real if an objective lapse of time exists. Then, he assumes that reality consists of layers of now that come to exist. In Savitt (2006) it is argued that this fact can lead, in principle, towards two different views on the metaphysics of time: *presentism* (the existence of the objects is relativized to the present, but they cease to exist after) and *possibilism* (the existence of the objects occurs in the present but it persists in the future). I maintain that Savitt’s analysis does not handle correctly Gödel’s views on the concepts of existence and time. Gödel has an absolute account of the concept of existence and he wants to determine the nature of time making saved his ontology. Moreover, Gödel could not use any metaphysical determi-

ual. In fact, if an individual X is determined by the set of his properties, then X can not inhabit two worlds w_1 and w_2 with different physical laws, since X in w_1 has at least different spatio-temporal properties in respect to X in w_2 .

nations of the objects in time, since the aim of his argument is to show the nature of time itself. Thus, Savitt's claim cannot be referred to Gödel's argument. In any case, the most critical point in Gödel's argument is that to infer the impossibility of a global time function in the R-worlds, which leads to an antirealist account of time in the R-worlds and in our universe, too.

Many attempts to understand the conclusion of this argument have been presented and some of them have been focused on the Gödelian ontology, notably on the role of the concept of "possible world". Notice that in 1949 there was no Kripke semantics for modal logic, but only a semantics for Lewis' modal logic system S_5 due by Carnap with the concept of "state description" (Carnap 1947). A state description is a class of sentences, which contains for every atomic sentence either this sentence or its negation, but not both, and no other sentences such that the *intension* of a term is a function from state descriptions to objects, while a predicate is a function from state descriptions to sets of objects. Thus, the concept of (logical) "necessity" is defined as true in every state description, while the concept of "possible" is defined as true at least in one state description. Notice that Carnap's framework does not deal with the different accessibility relations among state descriptions¹⁵ (or possible worlds) as we do in Kripke's frames for modal logic. Hence, the issue of the accessibility among different worlds was still very problematic in 1949, when Gödel wrote his argument for the ideality on time.

4. Possible worlds. Ontology and cognition

Jaakko Hintikka claims that Gödel maintains a "one-world assumption", since his concept of truth does not refer to the concept of "true in all possible worlds", but "true in the actual world". He writes that "one central idea of Leibniz's was never taken up by Gödel: the idea of possible worlds. This is one of the many indications of Gödel's actualism" (Hintikka 2000, p. 47; see also Hintikka 1998). Hence, logical and abstract entities are placed in an abstract region of the actual world, not in different possible worlds. Our epistemic access to this abstract region is provided by the mathematical intuition, which extends the powers of ordinary sensible perception. Thus, if the domain of discourse is our single world, then there is a collapse of modalities, viz. what is possible is necessary and vice versa.

¹⁵ We can say that a state description is the syntactical counterpart of the concept of "possible world" (or of Wittgenstein's concept of "state of affairs") in the modal system S_5 .

Van Atten (2001) criticises Hintikka's interpretation, since he notices that Gödel uses a possible-world reasoning in the argument for the "ideality of time" in one of his essays on the rotating universes. It seems that Gödel is assuming that the concept of possibility involved in his above argument is the *logical* one, which allows for an Euclidean relation as in Lewis' modal system S_5 . Notice that, in his argument about the ideality of time, the concept of *physical* possibility occurs, while it seems that in point 10 of the argument the concept of logical possibility is involved. Writes Gödel:

The mere compatibility with the laws of nature of worlds in which there is no distinguished absolute time, and, therefore, no objective lapse of time can exist, throws some light on the meaning of the time also in those worlds in which an absolute time can be defined (1949b, p. 562).¹⁶

Van Atten maintains that there is no collapse between *possibility* and *necessity* in Gödel's modal views, but between the concept of *possibility* and the concept of *actuality*.¹⁷ Namely, if something is possible, then it is actual, i.e., it can occur in our world.

I argue that we need to consider the absolute account of the Gödelian concept of existence again: "The concept of existence [...] cannot be relativized without destroying its meaning completely" (1949b, p. 559). Hence, my view is that one cannot say (like Hintikka) neither that the concept of "possible world" does not play any role in Gödel nor that Gödel is an a holder of a mere actualism in ontology. My claim is that Gödel possesses the concept of "possible world", but the Gödelian possible worlds cannot be completely alternative to our actual world (in time), because of his account of the concept of "existence" and also because the accessibility relations among possible worlds did

¹⁶ For some Gödelian observations on the nature of time written in his philosophical notebooks, see the last chapter of Cassou-Nouguès (2007).

¹⁷ Van Atten notes that the philosopher Nicolai Hartmann held an ontological view similar to Gödel's one, in the case of the ideal objects of our experience (see Hartmann 1937). The Kantian views on necessity, possibility and actuality are very different. In fact, according to Kant in the Postulates of the Empirical Thought we have that: "What agrees (in terms of intuitions and concepts) with the formal conditions of experience is *possible*"; "That whose coherence with the actual is determined according to universal conditions of experience is *necessary*"; and "What coheres with the material conditions of experience (with sensation) is *actual*" (A218/B266, my italics). Thus, in the Kantian views on the modalities there is an interplay between the ontological features of reality and our structure of the cognition (and intuition). Moreover, the Kantian modalities are *regulative* aspects of reality, while, according to Hartmann, modalities are the *constitutive* aspects of being.

not receive a proper formal treatment at the time. Namely, there cannot exist worlds which are completely independent of the actual world. Hence, if time turns out to be ideal in an R-world, then even in our actual world time cannot be real. Notice that this fact does not imply a form of actualism and does not coincide with the idea that the concept of possibility collapses into the concept of actuality, since the R-worlds are distinct from the actual world but their structure can influence our world.

A better understanding of Gödel's ontology can be achieved if one focuses on his ontology in the field of mathematics. As a matter of fact, the absolute Gödelian ontology entails that only static and atemporal objects can exist and this idea is completely different from the intuitionistic views on the nature of the mathematical objects which are dynamic and temporal. If an object is temporal, then it can be either *intratemporal* (namely, static and always existing) or *omnitemporal* (namely, an object comes to exist in a moment and ceases to exist in another moment). Van Atten observes that at first Husserl held an intratemporal view on the nature of the ideal objects, but after 1917 he realized that the temporal flow is a condition for the identity in their process of constitution.¹⁸ Husserl's later turned out to be not too much different from Brouwer's ones, in particular if one analyses the intuitionistic (law-like or free) choice sequences, they seem to be mathematical objects which behave in a very particular way, i.e., they are open only in respect to the future. Hence, one cannot say that choice sequences are completely *omnitemporal*, and, according to Gödel, they are not pure mathematical objects but objects which belong to applied mathematics, because of the role that time plays in their construction (see van Atten 2006). Namely, Gödel maintains that time cannot be a *constitutive* feature of the mathematical objects as in intuitionism. He wants to show with his argument of the ideality of time that our intuitive concept of time cannot be a *constitutive* element of the physical external world, too. Both claims are supported by his absolute ontology (which he calls "Platonism" or "conceptual realism"), in which time seems to play a relative role, while, from an epistemic point of view, he maintains that temporality is essentially connected with the mathematical intuition and the sense perception. In the 50's he writes:

The similarity between mathematical intuition and a physical sense is very striking. It is arbitrary to consider the proposition "This is red" an im-

¹⁸ This turn in Husserl's works coincides with a switch from the *static phenomenology* to the *genetic phenomenology*.

mediate datum, but not so to consider the proposition expressing *modus ponens* or complete induction (or perhaps some simpler proposition from which the latter follows). For the difference, as far as it is relevant here, consists solely in the fact that in the first case a relationship between a concept and a particular object is perceived, while in the second case it is a relationship between concepts. [...] There is no substantial difference between mathematics and other sciences (1953/59, p. 359).¹⁹

And in another Gödelian essay we can read:

That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g. the idea of the object itself (1964, p. 271).²⁰

Thus, according to Gödel there is no difference – from a cognitive point of view – between the way in which we can grasp the meaning of the formal objects and the objects which we perceive in reality. Hence, there is also no cognitive distinction between the actual universe and his (abstract) possible R-worlds and that is why he can claim for the ideality of the time from the R-worlds to our actual world. In both worlds our Euclidean intuition is always the same and it allows to interpret the non-Euclidean worlds in accordance with some geometrical translations such as the one from Klein. In any case, the importance of the (visual and Euclidean) mathematical intuition was firmly criticised by the neopositivists, who argued that not everything which can be *conceived* has a visual and intuitive counterpart as in the case of a space with *n*-dimensions or a chiliagon.

5. Concluding remarks

The relation between ontology and cognition is the fundamental point of the Gödelian views on time. I have showed in the previous sections that it is not too easy to conciliate the absoluteness of Gödel's timeless ontology with the temporality of his idea of cognition and intuition. Moreover, his subjectivist (and

¹⁹ The quotation is taken from the fifth version of the essay “Is Mathematics a Syntax of Language?”; the second and the sixth version of the essay are published in Rodríguez-Consuegra (1995).

²⁰ This quote shows also Gödel's critics towards the associationist models of mind.

incomplete) reading of Kant's works does not provide any systematic framework within which to handle the problem of the nature of time, which can be ideal in the R-worlds but there is no evidence to support that such kind of ideality holds in our world in time. It is true that there is the *logical possibility* that if time is ideal in an R-world, then it is ideal in our world, even if the R-worlds are coherent with the theory of relativity. This means that not all the solutions of Einstein's field equations are *physically possible*, but there are some solutions, like the ones which allow for the R-worlds, which are just *logically possible*. One of the many reasonable ways to discern the solutions which are logically possible from the ones which are physically possible can be to consider all the solutions which violate Mach's principle as not physically possible, but only logically possible.²¹ In this way, the Kantian idea of causality as a relational *magnitudo*²² (or interaction between substances) of the third Analogy of Experience can be vindicated as a concept which turns out to be particularly connected with the subsequent Mach's principle. Thus, the Kantian causality is a heuristic principle either in the modern theory of relativity and in the structure of human cognition. Kant maintains in the third Analogy that each object is considered to be the cause (as well as the effect) of the specific determinations of all the things perceived as simultaneous along with it (Longuenesse 1998, chapter 11). But this interaction between objects is Mach's principle *in nuce* and this is a very important connection between Kant and Mach (and the early formulations of the theory of relativity) on this point.²³

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²¹ See Horák (1979).

²² Kant observes that the knowledge of the *magnitudo* of the universe can only be *conditionally* analyzed and grasped also because we cannot intuit an infinitely large *magnitudo*. He also states: "we cannot therefore say anything at all in regard to the magnitude of the world, not even that there is in it a regress *in infinitum*. All that we can do is to seek for the concept of its magnitude according to the rule which determines the empirical regress in it. This rule says no more than that however far we may have attained in the series of empirical conditions, we should never assume an absolute limit, but should subordinate every appearance, as conditioned, to another as its condition, and that we must advance to this condition" (Kant [1781] 1965, B547-548).

²³ I am grateful to Arcangelo Rossi, Michiel van Lambalgen, Carlo Dalla Pozza and Mario Castellana for useful comments and suggestions.

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