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Comparing Two Views of Comparison: Peña and Casari on Vagueness and Comparatives

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ABSTRACT. This paper aims at a comparative assessment of two degree-theoretical views of vagueness and comparison – Ettore Casari's comparative logic and Lorenzo Peña's transitive logic. Although both approaches cope better than most rival theories with the thorny challenges posed by such issues, in the author's opinion Casari's perspective seems superior under a number of respects.

KEYWORDS: vagueness; comparatives; sorites paradox; comparative logic; transitive logic; Casari, Ettore; Peña, Lorenzo.

1. Introduction

After having been working for a number of years on Casari's comparative logic and its applications to the issues of vagueness and comparison, I have recently come across a closely related approach to these problems which has been developed over the last decades, unbeknownst to me, by the Spanish philosopher Lorenzo Peña and his coworkers. Both research streams started at

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about the same time (the beginnings of Peña's work on transitive logic date back to the mid-seventies, while Casari's earliest published article on comparative logic was originally presented at a conference in 1979; see Casari 1981) but have hitherto evolved – to the best of my knowledge and as far as I can infer from the lack of mutual references in the writings of both schools – in a completely independent way. Yet, besides sharing a common underlying view of the phenomena under investigation, these approaches seem to show striking similarities both in their general philosophical outlooks and in their technical underpinnings, as well as a number of differences. It seems therefore useful to have a closer look at such similarities and differences. The aim of the present paper will be, if you let me put it this way, to compare the above-mentioned views of comparison.

I will not try to give a self-contained account either of comparative logic or of transitive logic, nor will I aim at a detailed exposition either of Casari's or of Peña's views on gradability and comparison: the interested reader should consult Casari (1989; 1997), Paoli (1999; 2003), Peña (1984; 1987a; 1987b; 1990; 1992; 1993; 1995; 1996), and Vásconez, Peña (1996) for more systematic information of this kind. Rather, I will discuss in a quite haphazard way some specific philosophical features of both approaches, trying to single out where they agree with each other and where they are at variance. Moreover, the reader is warned that my discussion will not be symmetric and neutral, but rather biased in favour of comparative logic (the term "bias" is taken here, of course, in its positive meaning of a rationally and critically supported inclination, which must not be confused with prejudice!). Due to this, I will mainly focus on a series of remarks on – often also objections to – Peña's approach as seen against the background provided by Casari's system.

Finally, let me frankly confess that my knowledge of the work of Peña and his disciples is utterly partial, as it mainly results from the items referenced in the bibliography below, for many of which I have been kindly provided with copies by the author himself, and from some stimulating conversations with Peña's coworkers Marcelo Vásconez and Txetxu Ausín (whom, by the way, I thank for stirring my interest into transitive logic and its underlying philosophy). Whether in Peña's extensive bibliography there are further writings which could substantially modify the discussion provided below, I do not dare to say.

2. The shared general framework

For a start, I will try to underscore the most apparent similarities between the above-mentioned perspectives. Before doing this, however, some terminological remarks are in order. Whenever one's talk is about vagueness lexical quarrels are quite likely to arise, since the very definition of such a term is a contentious matter. In particular, it seems as though authors belonging to the streams discussed here use this term in different meanings. At the present stage, however, I propose to stay with a rather inexpressive definition: let us call "vagueness" the phenomenon which gives rise to slippery slope arguments of soritical type, whatever its nature may be. We will see below to what extent the comparative-logical and the transitive-logical approaches make different uses of this word; until then, I beg the reader to accept my verbal convention.

The main similarities between both approaches can be summarized as follows:

A) Ontological view of vagueness. According to both perspectives, vagueness is neither an epistemic phenomenon of ignorance, as claimed by Williamson and other epistemicists, nor a semantic phenomenon of ambiguity, as maintained by supervaluationists like Fine; rather, it is an *ontological* phenomenon. Each vague predicate does not admit of borderline cases of application because it denotes a sharp property of whose extension we are in principle ignorant, or because it ambiguosly refers to several sharp properties, but because the unique property it refers to is itself *fuzzy*, in that it applies to some objects only to some extent. This stance is explicitly upheld in the writings of Peña:

> La aplicación de predicados difusos no se debe a alguna aberración de nuestro pensamiento o de nuestro lenguaje, sino que está basada en el carácter objetivamente difuso de ciertos cúmulos o propiedades, a saber aquellos que abarcan a alguno de sus respectivos miembros en una medida no total (1996, p. 146).

Casari is not just as outspoken on this aspect, but I think that the ontological view of vagueness is the one which best accords with his degree-theoretical approach to the issue.

B) Degree-theoretical attitude. According to both perspectives, thus, the possession of a property by an object, or the membership of an element in a set, is not an all-or-nothing issue, but a matter of *degree*. Degrees of memberships are paralleled, on the level of sentences, by *degrees of truth*: if it is a matter of degree whether or not John belongs to the set of tall people, it cannot be but a matter of degree whether or not the sentence "John is tall" is true. Also, a proper logical treatment of such kind of sentences demands that degrees of truth be *infinitely* *many*. Of these degrees, some will be positive (true) and some negative (false) – and here similarities on this aspect come to a sudden stop, as we will see.

C) Reductionist approach to comparatives. An adequate theory of adjectival comparison must be wide-ranging enough as to make room at least for cross-comparisons of the form

(1) *x* is at most as *P* as *y* is *Q*,

and the likes where "at most as... as" is replaced by "at least as... as", "more... than", "less... than". But this can be accomplished, together with a reduction of comparative adjectives to the corresponding positive adjectives, by resorting to the apparatus of truth degrees. In fact, the following *reduction principle* is endorsed within both perspectives:

(2) A sentence of the form specified in (1) is true iff "*x* is *P*" implies "*y* is *Q*"

where implication is an inherently comparative notion whose behaviour is governed by the following *simplification principle*:

(3) *A* implies *B* iff the truth degree of *A* is smaller than or equal to the truth degree of *B*.[1]

(D) Real fuzziness principle. All the features discussed under the previous headings are common not only to Casari's and Peña's approaches, but virtually to any fuzzy, degree-theoretical perspective on vagueness and comparison (where "fuzzy" is meant here in a broad, non-technical sense). Our two approaches, however, are quite serious about fuzziness, which is really taken at face value. Remember that, according to the received view on the subject, vague predicates are *tolerant* – in other words, it is possible for *x* and *y* to differ (if only very slightly) in their degrees of possession of the property *P* while the truth values of the sentences "*x* is *P*" and "*y* is *P*" do not differ at all (Wright 1975). As I illustrated in Paoli (2003), the standard fuzzy approaches, like the one based on Łukasiewicz logic, reject this assumption within the area of borderline cases, but not outside it – i.e., as far as definitely positive or negative *P*-cases are concerned. A really fuzzy semantics of

[1] A principle encompassing our simplification and reduction principles is called *principio de desincrustación* in Peña (1987b, p. 342).

vagueness and comparison, instead, must take into account all the relevant differences:

> Any difference, however small, in the possession of a property by an object does affect the justice with which the corresponding predicate can be applied to it (Vásconez 2002, p. 39).

3. Vagueness, fuzziness, gradability

So much for common ground. Let us now come to review the main points of disagreement between the theories under scrutiny.

We hinted above at a terminological discrepancy concerning the term "vagueness". According to Peña, in fact, vagueness is a pragmatic phenomenon that plays no role in soritical paradoxes – where it is the property of *gradability*, or of coming in degrees, which is really at issue:

> Vagueness is not the same as graduality. A statement may be vague because of pragmatic considerations concerning the context of utterance: the utterer is expected to convey more specific information and instead he contents himself with general remarks [...]. Terms can be said to be vague in so far as they are used in vague statements. Yet on their own they are not vague. "Tall" is not vague. It is a term denoting a property which comes in degrees. The issue is not vagueness but graduality (1993, pp. 403-404).

About such a pronouncement I have two comments to do. On the one hand, this usage of the word "vague" does not tally with the established usage to be found in substantial portions of the current scientific literature on the argument, where the phenomenon just described by Peña – failure to provide information which is specific enough – is labelled *generality* and is sharply distinguished from vagueness (see e.g. Sorensen 2002; Tye 1994). It must be said that Peña advances some appealing arguments to the effect that his terminology is more consistent with the everyday meaning of the term (see e.g. Peña 1996; Vásconez, Peña 1996); however, I believe that consistency with established philosophical use gives me at least some right to employ the word "vagueness" whenever Peña would resort to "fuzziness" (which I prefer to avoid in this case since it is by now too loaded with technical connotations: over the years, it has come to refer to a *particular theoretical approach* to vagueness, rather than to vagueness in itself).[2]

[2] At the beginning of Section 5 I will expand a little bit on this issue.

On the other hand, and apart from such terminological controversies, I have a more conceptual kind of qualm regarding the identification of vagueness (Peña's "fuzziness") with gradability. I argued elsewhere (Paoli 1999) that gradability is a necessary, but not a sufficient condition for vagueness.[3] In fact, there are predicates – like "acid" as a predicate of chemical substances, or "acute" as a predicate of angles – which seem to come in degrees and admit a nontrivial comparative: it seems perfectly all right to say, e.g. that a substance with PH3 is more acid than a substance with PH5, or that a 25° angle is more acute than a 65° one. Nonetheless, these predicates are not vague at all and can by no means give rise to sorites of any kind, because there are sharp cut-off points separating their domains of application from the domains of their antonyms (in our examples: PH7 for "acid", 90° for "acute").[4]

Examples like these lead us to surmise that vagueness is not just gradability, but a *special* kind of gradability, arising whenever a given predicate P admits of borderline cases of application. Casari's semantics – where one allows for *intermediate* degrees of truth, which are neither definitely true nor definitely false, as well as for degrees of definite truth and falsity – yields a satisfactory treatment of the distinction between gradability and vagueness. Generally speaking, a one-place property can be identified with a function from individuals of an appropriate domain to degrees of truth. In particular, a sharp gradable property (like acidity) is a function which takes up only positive ("true") or negative ("false") degrees, but never intermediate ones; whereas a vague property is a function which can take up positive, as well as negative and intermediate degrees. Whether such a distinction can be reproduced in the semantics for transitive logics, it is unclear to me.

4. The rules of endorsement and maximality

Although several different systems coexist in the family of transitive logics, they all can be endowed with an infinite characteristic matrix whose set of truth values is the closed real unit interval [0,1] and whose set of designated values is the semiopen interval]0,1] – i.e., the whole set of truth degrees ex-

[3] Other authors, like Bierwisch (1989), claim that it is not even a necessary condition, but their contentions need not concern us here.

[4] Interestingly enough, Engel – an author quoted by Peña as being somewhat sympathetic with his approach – discusses such examples (Engel 1989), to which however Peña does not seem to pay special attention.

cept 0, absolute falsity (differences among logics in the family mainly amount, if I am not mistaken, to differences in expressive power). Degrees in the open interval]0,1[– all the truth degrees except absolute truth and absolute falsity – are to some extent both true and false. Also, Peña espouses a rather uncontroversial (in degree-theoretical terms) semantics for negation, according to which the amount of truth in the negation of *A* increases as the amount of falsity in *A* increases, and vice versa. Examples from natural language clearly show that intermediate degrees are inhabited, i.e. there are sentences that can be assigned such degrees – hence, also their negations will be assigned intermediate truth degrees. Given the above choice of designated degrees, this means that there must be true contradictions (where it is essential to remark that "true", here, means "true to some extent").

From a philosophical viewpoint, this somewhat unusual selection of designated values (unusual at least as far as fuzzy logics are concerned) is bolstered by an assumption which Peña terms *rule of endorsement* (*regla de apencamiento*) and which is formulated and justified in the following terms:

> La *regla de apencamiento* nos permite pasar de "*p* es en alguna medida verdadero" a la conclusion de que "*p* es verdadero". Esta regla ha venido reconocida como legitima en la tradición logica y filosofica. La base de ese reconocimiento es que no puede ocurrir que la conclusion sea totalmente falsa, en el supuesto de que la premisa sea, en uno u otro grado, verdadera (Vásconez, Peña 1996).

> Since what is required in order for a statement to be right is nothing else but its being true (just true, without further qualifications), and any statement is (to some extent or other, however small) true unless it's wholly untrue, we can safely state any sentence provided we are convinced it's not altogether false (Peña 1984).

Upholding the rule of endorsement goes hand in hand with refusing the incompatible assumption named *rule of maximality*:

> What definitely must be waived is the maximality rule, viz.: $p \vdash Hp$.[5] [...] The purported rationale for it is that nothing can be rightly asseverated unless it's quite true, true without mixture of falsity. So, "true"' *tout court* would be equivalent to "utterly true". But to my mind such a reason doesn't carry conviction.

[5] "*Hp*" means "it is definitely the case that *p*" in Peña's notation, and is true just in case $v(p) = 1$.

> For, when we assert some sentence, we to be sure regard it as true; but why on earth should we regard it as wholly true? When I say that I'm hungy, I'm not saying that I'm altogether hungry, but just hungry [...]. The maximality rule thus lends to relinquishing what is of importance in fuzzy logic, namely: truth-nuances, in virtue of which not all that is true is entirely so (Peña 1984).

The standpoint of comparative logic is, here, completely different. Not only is the choice of the interval [0,1] as a system of truth degrees rejected, for reasons to be seen in the next Sections, but also contradictory degrees and the rule of endorsement are rebutted, while a properly understood maximality rule is accepted. In fact, comparative logic acknowledges that some sentences containing vague predicates can be approximately true and at the same time approximately false, but equates truth *tout court* not with approximate truth, but with *definite* truth. The important difference with respect to the standard fuzzy approaches, which permits a satisfactory account of vagueness and comparison, is the idea that *not only approximate truth (falsity), but also definite truth and definite falsity come in degrees*.

In my view, it is important to keep in mind some conceptual distinctions that are somehow overlooked by Peña. In particular, we must distinguish two senses of "possessing a property" and just as many senses of "definite truth". "Possessing a property *P*" can mean either of two different things for an individual *a*:

(a) That *a* has *P* to some non-null, however small, degree;
(b) That *a* has *P* to as high a degree as to warrant applicability of the predicate "*P*" to the name "*a*". It seems to me reasonable enough to maintain that the sentence *Pa* must be considered true just in case *a* has *P* in this second sense.

Likewise, "being definitely true" can mean either of two different things for a sentence *A*:

(c) That *A* is true "without mixture of falsity", i.e. it is not even approximately false;
(d) That no sentence *B* can be strictly truer than *A*.

In my opinion, the rule of endorsement covertly presupposes the identity of (a) of (b); but this causes no end of trouble, as the next example will show. On the other side, I concede that the rule of maximality is blatantly invalid if "definitely true" is taken in sense (d) – more than that, I even deny that such a notion makes any sense at all – yet think that there is nothing to blame in it if "definitely true" is correctly understood in sense (c).

With an eye on these distinctions, I will now argue that accepting the rule of endorsement would lead us in any case to unpalatable conclusions: that is, either to a clash with ordinary linguistic usage and with some naive intuitions concerning the meaning of predicates, or else to a violation of the real fuzziness principle and the semantics of comparison, i.e. of what we identified as two of the strong points of both Casari's and Peña's approaches.

Let John be an European adult who is just 1.20m tall. Should we say that John possesses to some extent the property of tallness? In other words, should we say that it is to some extent true that John is tall? Either way we go, we are in trouble. Suppose we do. Then, by the rule of endorsement, it is true that John is tall. Well, this would seem to contradict linguistic practice: nobody in his right mind would agree that John is tall (for an European adult). If the envisaged measure of 1.20 still seems controversial, just tweak the example to taste replacing it with as small a measure as you wish.

On the other hand, one might say that John completely lacks the property of tallness – after all, he's not even remotely a borderline case for a tall person. But this is at odds with our beloved real fuzziness principle, as well as with the account of comparison seen above. For suppose that Bill is 1.18m tall. Bill is strictly less tall than John, and thus should possess the property of tallness to a strictly smaller degree than John does. But how can that be, if John completely lacks the property?

Comparative logic provides a cheap way out from such puzzles. Under the given circumstances, it would seem reasonable to most of us to say that "John is tall" is definitely false, i.e. false and not even approximately true. However, since definite falsity also comes in degrees, "Bill is tall" will have a strictly falser degree of truth. This seems to reconcile our everyday understanding of sentences like the above with the principle according to which even minute differences in the possession of an attribute should be mirrored in one's semantics.

5. The structure of truth degrees

Comparative logic is not a fuzzy logic in the technical, narrow sense of the word, since it has no characteristic matrix semantics on the real unit interval [0,1]. I believe that the choice of such interval as a system of truth degrees carries two main shortcomings: this set, in fact, is *bounded* and *linearly ordered*.

I am going to be very cursory as regards these questions, since I discussed them rather extensively elsewhere (Paoli 1999; 2003). I just recall that a bounded set of truth values does not allow a satisfactory reconstruction of comparative sentences at least in three cases: when cases of definite applicability of a predicate are at issue, where comparative constructions are nested, and when the predicates involved are gradable but sharp. On the other hand, linear ordering seems too rigid a constraint for comparisons involving evaluative predicates and semantically anomalous comparative sentences.

To be sure, Peña occasionally seems to acknowledge the insufficiency of the linearity constraint (see e.g. Peña 1995). His hints at a "tensorial semantics" could be seen as a first attempt to tackle the issues just alluded to.

6. The semantics of logical connectives

So far we have not been concerned with the logical structure of sentences as respectively analyzed by comparative and transitive logics. It is about time we come deeper into this interesting topic.

As I fleetingly recalled above, transitive logics differ from one another mainly in having more or less rich stocks of logical constants. Here, my chief interest will lie in *propositional* logics – thus, I will leave aside issues related to quantification, even though, of course, I am most ready to acknowledge their crucial role for a proper reconstruction of the domains of discourse we are treating. The connectives of transitive logics which I intend to discuss (or, in any case, which I need for my discussion) are:

- weak negation (symbolized as N), which can be evaluated in a number of ways; an especially convenient one, also consistent with current fuzzy semantical practice, is taking $v(NA) = 1\text{-}v(A)$;
- strong negation (symbolized as $\neg$, which should be read as "It is absolutely not the case that..." and is thereby a sort of Stonean negation (Ovchinnikov 1983): $v(\neg A) = 1$ if $v(A) = 0$, $v(\neg A) = 0$ otherwise;
- conjunction (symbolized as $\wedge$, which obeys the clause $v(A \wedge B) = \min(v(A),v(B))$;
- disjunction (symbolized as $\vee$, which obeys the clause $v(A \vee B) = \max(v(A),v(B))$;
- the conditional (symbolized as $\supset$, definable in terms of strong negation and disjunction: $v(A \supset B) = v(\neg A \vee B)$;

– finally, implication (symbolized as $\rightarrow$), whose evaluation clause is given by $v(A \rightarrow B) = \frac{1}{2}$ if $v(A) \leq v(B)$, $v(A \rightarrow B) = 0$ otherwise.

From the above clauses, it is easy to see that while *overcontradictions* (sentences of the form $A \wedge \neg A$) are always false, *simple contradictions* (sentences of the form $A \wedge NA$) may well be true. This, however, does not amount to a waiver of the law of noncontradiction, which – as it is immediate to check on the basis of the given semantics – is indeed valid. Likewise, the law of excluded middle is also valid even if the negation therein is weak negation:

> No hay por qué sacrificar el PTE [the excluded middle] en presencia de lo difuso. Todo lo contrario, de hecho, se puede probar el PTE [...] no sólo lo difuso no se opone al PTE, sino que lo entraña. [...] El PTE es verdadero y falso a la vez. Nada de extraño (Vásconez, Peña 1996).

Peña remarks that, if one accepts involutivity of negation and De Morgan laws, the excluded middle and the principle of noncontradiction stand or fall together. Therefore, any argument for the former also counts as an argument for the latter. Here is a possible argument in defence of excluded middle, to some extent independent of the particular valuation clauses for connectives chosen above (see Peña 1984).

If any sentence of the form $Pa \vee NPa$ has to be a counterexample to the excluded middle, it must perforce be one in which a is a borderline case for a P; in fact, if a is either a definitely positive or a definitely negative P-case, that instance of the excluded middle will be unquestionably true. However, if a is a borderline case for a P, both Pa and NPa will be true to some extent or other – hence true, by the rule of endorsement. By adjunction, $Pa \wedge NPa$ will be true; then, so will be Pa by simplification and $Pa \vee NPa$ by addition. If there are no counterexamples of the envisaged form, there seems to be no reason why there should be counterexamples of a more complex logical form. Thus, there are no counterexamples to the excluded middle.

It is not hard to see where the previous argument breaks down, according to the comparative-logical perspective: the mentioned application of the rule of endorsement, in fact, does not appear to be warranted. Borderline cases for vague properties do indeed provide counterexamples to the excluded middle in that both disjuncts are only approximately true, but not definitely true; hence there is no reason to suppose that the disjunction itself be definitely true.

Of course, the comparative logician now owes an explanation of the widespread credit given to such a law by the community of logicians. Why is the excluded middle generally regarded as valid? Here is a possible answer.

Its good reputation hinges on an unappreciated equivocation lurking behind the word “or”. According to comparative logic, in fact, there are at least two kinds of inclusive disjunction – which collapse onto each other in classical logic, but are to be kept distinct in a degree-theoretical perspective. The first one is what we might call a *parallel* disjunction: each disjunct is evaluated separately in order to ascertain its degree of truth, and the whole disjunction is evaluated as true if at least one disjunct turns out to have a positive (“true”) degree of truth. On the other side, we have a *comparative* disjunction: the degrees of truth of the disjuncts are not assessed independently, but compared to each other, and the whole disjunction comes out true just in case the amount of falsity in each disjunct does not exceed the amount of truth in the other; in other words, just in case each disjunct is at most as false as the other is true.

Therefore, even if negation is taken to mean weak negation and not strong negation, not all ambiguity is dissolved: the principle of excluded middle can still be understood in two different ways. The *comparative excluded middle* is undoubtedly valid, because it falls straight out of harmless semantical stipulations concerning negation that each one of A, NA is exactly as false as the other one is true. Nonetheless, the *parallel excluded middle* could admit of counterexamples, because both disjuncts might well fail, on their own, to meet the standards of truth. The outcome of all this is that there is a sense in which the excluded middle is indeed valid, whereby its popularity among logicians can be at least partly accounted for.

Another less than satisfactory aspect of transitive semantics is the non-gradability of implicational sentences, which can receive just *two* degrees of truth – a designated and an undesignated one. Either an implicational sentence is half-true, or else it is absolutely false. I argued elsewhere (Paoli 1999; 2003) that degree-theoretical approaches which endorse our principles of reduction and simplification but assign the same degree of truth to all true implicational sentences (for example, approaches based on Łukasiewicz logic) are bound to fail whenever nested comparative sentences are at issue.

Beyond that, this feature of Peña’s theory seems to contravene once again the real fuzziness maxim. In fact, let John be 1.70m tall, Bill be 1.72m tall, and Rick be 2.00m tall. It does no violence to our customary linguistic habits to say that Bill is only *slightly* taller than John, whereas Rick is *much* taller than John. The use of hedges points toward a property, that of being taller than John, that Bill and Rick possess to different degrees – and it falls out of the meanings of those hedges that Rick possesses it to a *higher* degree than Bill does. All these ruminations can be rephrased without undergoing much conceptual change (maybe only at the expense of intuitive perspicuity) if we replace “is taller than...” by “is at least as tall as...”. However, given our simplification and reduction principles, “Bill is at least as tall as John” means nothing but “That John is tall implies that Bill is tall”, and the same applies to the sentence where “Bill” is replaced by “Rick”. Both such sentences are true implicational sentences, hence they get value ½. But then transitive semantics fails to reflect the difference in the respective degrees of possession of the property “being at least as tall as John” by Bill and Rick.

7. The sorites

Any purported theory of vagueness cannot claim to be tenable unless it provides a satisfactory account of the most debated puzzle in this whole area, the sorites paradox. Indeed, Peña attempts such an explanation. Let us now try to give the gist of it.

Consider the following version of the paradox. Suppose, for the sake of simplicity, that the sole measure of one’s wealth is given by the amount of currency in one’s bank account. Whoever has just one dollar in his account is poor. Moreover, suppose the properties of two people differ just by one dollar: since such a gap is too slight to make any difference as regards the application of the predicate “poor”, this means that either the people at issue are both poor or they are both non-poor. From these premisses it follows that whoever has two dollars in his account is poor; but also that whoever has three dollars is such... Sliding down the slippery slope, we are little by little drawn to the unescapable, but absurd conclusion that whoever has 100,000,000 dollars in his account is poor.

If we let $P(n)$ stand for “Whoever has n dollars in his account is poor”, the argument rests on a categorical premiss which can be represented as

(4) $P(1)$

and, for each n, on a disjunctive premiss of the form

(5) Either it is not the case that $P(n)$, or $P(n + 1)$.[6]

I purposely refrained from stating (5) in a formalized guise. Indeed, the first step towards a proper understanding of the argument amounts to determining the

[6] This is exactly what our above formulation says, if pruned of uncontroversial information.

actual logical form of (5). For example: does "it is not the case that..." express a weak or a strong negation? According to Peña, the negation at issue cannot be strong: suppose, in fact, that n is such that $P(n + 1)$ is absolutely false, but $P(n)$ is – if only to an extremely small degree – true. Then $\neg P(n) \vee P(n + 1)$ is false, and the argument is unsound. On the other hand, if the negation is weak, each disjunctive premiss must be true: the only possible way for $NP(n) \vee P(n + 1)$ to be false is just in case $P(n)$ is absolutely true, while $P(n + 1)$ is absolutely false – which cannot be, since adjacent objects in a soritical sequence cannot differ so much in their respective possessions of the relevant property as to make such an assignment of values possible. Therefore, (5) must be formalized using a weak negation (and a parallel disjunction). Yet Peña claims that the argument, if so understood, is *invalid* because disjunctive syllogism does not hold for weak negation. Suppose in fact that n is a borderline case for a P; then both $P(n)$ and $NP(n)$ will be true to some degree, and the inference from $P(n)$ and $NP(n) \vee P(n + 1)$ to $P(n + 1)$ will be unwarranted ($P(n + 1)$, for all that we know, might be absolutely false). Summing up:

> Los sorites se resuelven admitiendo la premisa mayor solo en la versión disyuntiva [...]: o uno de dos términos consecutivos en la cadena carece de la propiedad en cuestión, o el siguiente la tiene. Hay que rechazar la formulación condicional – aquella según la cual, si el uno la tiene, el otro también. No valiendo, en general, el silogismo disyuntivo, no nos veamos llevados a la conclusión desastrosa de que todo posee la propiedad en cuestion (Peña 1996, p. 146).

We just saw that, in the opinion of Peña, the connectives involved in the disjunctive premisses of the sorites are weak negation and *parallel* disjunction. Although I agree with the claim about negation, I believe that understanding disjunction as a parallel connective misrepresents the logical structure of the argument. Suppose in fact that someone, at first sight quite plausibly, were to contend that the disjunctive assumptions of this piece of reasoning are true. Would such a contention be based on a separate evaluation of each disjunct? Quite otherwise: the very fact that our proponent maintains that *every* disjunctive premiss is true, independently of the value of n, implies that she is deeming each disjunction true neither out of the truth of its negated disjunct, nor out of the truth of its atomic one: rather, such an assessment is based on a *comparison* of their respective degrees of truth. What gives the argument its bite is the fact that each disjunctive premiss is somehow meant as instantiating the principle of tolerance: all it says is that $P(n)$, whatever its degree of truth may be, is just as true as $P(n + 1)$ is, whence $NP(n)$ is just as false as $P(n + 1)$ is true – and thus, in particular, is at most as false as $P(n + 1)$ is true. The disjunction we are encountering, therefore, is a comparative disjunction and not a parallel one.

Does Peña's system possess the expressive means to define a comparative disjunction? Very much so indeed: such a disjunction is definable as $NA \to B$. And would the paradox be solved all the same if its premisses were so rephrased? From a purely technical viewpoint, it would. Each premiss would simply be equivalent to $P(n) \to P(n + 1)$ and, since the degree of truth of $P(n)$ would have to be strictly greater than the degree of truth of $P(n + 1)$ by the real fuzziness principle, Peña's semantics for implication would dictate that $P(n) \to P(n + 1)$ be absolutely false. The argument, in sum, would be valid – disjunctive syllogism holds for comparative disjunction – but unsound.

However, a proper solution to a paradox must not only explain what goes wrong in the chain of deductions, but also account for the *prima facie* appeal of the argument. If the disjunctive premisses are evaluated as absolutely false, the task is not accomplished: why do we feel so tempted to endorse what, after all, is just an absolute falsity? Remark that pragmatic grounds can be of no avail here: were we to claim that the disjunctive premisses are false but "highly assertable", according to some extra-logical notion of assertability, why not stay with classical logic and apply our pragmatic theory to it instead?

The solution provided by comparative logic, on the other hand, is along the lines of the standard degree-theoretical replies to the sorites: the disjunctive premisses are not definitely true, but fall barely short of definite truth. This explains their appeal, while leaving room for a dismissal of the argument as unsound. The additional gain, with respect to the standard approach, is that the recourse to the flexible framework of comparative logic makes it possible to evaluate all disjunctive premisses in the same way and to properly account for the *uniformity* of any soritical accumulative process (see Paoli 2003 for a more detailed discussion).

8. Conclusion

Let me conclude with a couple of remarks concerning the paraconsistent character of the logics under examination, and their overall bearings on philosophy.

Transitive logics are both paraconsistent and dialethic, i.e. not only they rebut the *ex absurdo quodlibet* but also they contain some contradictions as theses. On the other hand, comparative logic is not a dialethic logic but is paraconsistent – it contains non-trivial and well-motivated inconsistent extensions, such as Abelian logic.

Finally, I would like to conjecture that some of the differences between transitive and comparative logics could perhaps be explained in terms of the different interests of their respective founders, which probably determined a divergence in the intended philosophical applications of the logics themselves. Comparative logic originated, in Casari's intentions, as an attempt to reconstruct the theory of comparison advanced by Aristotle in his *Topics* and elsewhere, but later such historical motivations were outdone by purely mathematical stimuli on the one side, and by the desire to offer significant contributions to the semantics of natural language on the other. If I am not mistaken, transitive logics are more conspicuosly driven toward applications to other branches of philosophy – like metaphysics, theology, philosophy of law or epistemology – which at least so far have not been the primary concerns for comparative logicians.

It may well be possible that comparative logic cannot withstand objections arising from such areas of philosophical debate. What I would like to stress, however, is that neither Casari nor I have ever taken comparative logic to be an all-purpose logic which is appropriate to solve each and every philosophical problem. Rather, comparative logic should be understood as a *task-oriented logic*: a framework which perhaps could prove more adequate than its rival approaches to formally account for a limited fragment of natural language, which is however rich enough to contain gradable and vague predicates – as well as a good deal of adjectival and nominal comparative constructions.

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REFERENCES

BIERWISCH, M. 1989: "The Semantics of Gradation", in M. Bierwisch and E. Lang (eds.), *Dimensional Adjectives*, Berlin: Springer, pp. 71-262.

CASARI, E. 1981: "Remarks on Comparison and Superlation", in *Atti del Convegno Nazionale di Logica 1979*, Napoli: Bibliopolis, pp. 261-271.

——— 1989: "Comparative Logics and Abelian *l*-groups", in A. Valentini *et al.* (eds.), *Logic Colloquium '88*, Amsterdam: North Holland, pp. 161-190.

——— 1997 "Conjoining and Disjoining on Different Levels", in M. L. Dalla Chiara *et al.* (eds.), *Logic and Scientific Methods*, Dordrecht: Kluwer, pp. 261-288.

ENGEL, E. 1989: "On Degrees", *Journal of Philosophy*, 86, pp. 23-37.

OVCHINNIKOV, S. V. 1983: "General Negation in Fuzzy Set Theory", *Journal of Mathematical Analysis and Applications*, 92, pp. 234-239.

PAOLI, F. 1999:"Comparative Logic as an Approach to Comparison in Natural Language", *Journal of Semantics*, 16, pp. 67-96.

——— 2003: "A Really Fuzzy Approach to the Sorites Paradox", *Synthese*, 134, pp. 363-387.

PEÑA, L. 1984: "Identity, Fuzziness and Noncontradiction", *Noûs*, 18, pp. 227-259.

——— 1987a: *Fundamentos de Ontología Dialéctica*, Madrid: Siglo XXI.

——— 1987b: "Contribución a la lógica de los comparativos", in C. M. Vide (ed.), *Lenguajes naturales y lenguajes formales*, Barcelona: University of Barcelona Press, pp. 335-350.

——— 1990: "Partial Truth, Fringes and Motion: Three Applications of a Contradictorial Logic", *Studies in Soviet Thought*, 39, pp. 283-312.

——— 1992: "Nuevas aplicaciones filosóficas de lógicas multivalentes", *Theoria*, 16-17-18, pp. 141-163.

——— 1993: "Review of Douglas Walton, *Slippery Slope Arguments*", *Pragmatics and Cognition*, 1, pp. 401-410.

——— 1995: "Lógicas multivalentes", *Enciclopedia IberoAmericana de Filosofía*, Madrid: Trotta-CSIC, pp. 323-349.

——— 1996: "Grados, franjas y líneas de demarcación", *Revista de Filosofía*, 9/16, pp. 121-149.

SORENSEN, R. 2002: "Vagueness", *Stanford Encyclopaedia of Philosophy*, http://plato.stanford.edu/entries/vagueness/vagueness.html.

TYE, M. 1994: "Vagueness: Welcome to the Quicksand", *Southern Journal of Philosophy*, 33 (Suppl.), pp. 2-22.

VÁSCONEZ, M. 2002: *The Sorites Paradox: General Survey*, Licentiate Thesis, Katholieke Universiteit Leuven.

VÁSCONEZ, M., PEÑA, L. 1996: "¿Qué es una ontología gradual?", *Agora*, 15, pp. 29-48.

WRIGHT, C. 1975: "On the Coherence of Vague Predicates", *Synthese*, 30, pp. 325-365.