REVIEW

JC BEALL and BAS C. VAN FRAASSEN
Possibilities and Paradox: An Introduction to Modal and Many-Valued Logic
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Once upon a time the phrase “non-classical logic” was roughly synonymous with “philosophical logic”. This widespread identification had only in part its roots in the fact that several non-classical logics – like modal, many-valued, or relevance logics – originally arose out of genuine philosophical motivations; just as decisive was the belief that it was possible to do non-classical logic while escaping at least some of the technical subtleties that the classical logician had to face. Once a new alternative to the received view had been devised, its pioneers were unavoidably compelled to start from scratch – and that meant they had to stay for a while at the *propositional* level, whereby they could spare themselves all the mathematical complications brought about by arithmetic, set theory, higher order logic and the like.

Later on, it became clear enough that the undoing of such knots was only postponed, not at all eliminated. As each new logic was acquiring ever more solid foundations, new problems emerged that required considerable mathematical sophistication to be properly tackled, and such a degree of complexity was usually greater than in the standard case, because it was not possible to rely on some simplifying assumptions inherent to the classical framework. As a result, the average paper or book on non-classical logics is, nowadays, as hard a read as most publications on classical logic – if not harder.

One is then favourably impressed whenever a book comes out – like Beall’s and van Fraassen’s *Possibilities and Paradox: An Introduction to Modal and Many-Valued Logic* – which tries to make these subjects accessible to the non-specialist and to underscore the philosophical contents and stimuli underlying each system or semantical framework

under discussion. Of course, there would be an easy way to accomplish this: just dodge the toughest issues and be content with a superficial smattering of the topic. Instead, Beall and van Fraassen go for the hard way. They do not refrain from entering into detail when necessary, but – except on rare occasions, of which more below – they always manage to keep the formalism to a minimum and at the same time to give a precise and highly informative account. The book is a real pleasure to read. Its threads are well-connected and well-balanced. The exercises are sensibly gauged and designed to stimulate both technical and conceptual ingenuity. Like Priest's *Introduction to Non-Classical Logic*[1], this volume can be an excellent companion for a first exploration of the fascinating logical territories that lie beyond the classical borders.

The exposition of Beall and van Fraassen is framed within a general view of logical pluralism which leaves some reason for disagreement. As it is fleetingly hinted at some places in the book, "logic derives from the structure of language, and […] therefore a plurality of logics will derive systematically from a plurality of different structured forms of discourse", although "the concept of premises validly implying a certain conclusion is general, not language specific" (p. ix). Here, it is essential to remark, "language" does not refer uniquely to the syntactical component – semantics play an essential role in the characterization of the notion. Briefly put, Beall and van Fraassen claim that differences among logics depend on differences in the underlying languages; the notion of validity is nonetheless the same across different logics, and each logic simply provides the catalogue of valid arguments for the language in which it is expressed.

My main objection against such a point of view is that it does not make logical pluralism – a worthwhile and justified perspective, in my opinion – immune from Quinean attacks based on meaning variance arguments. What guarantee are we offered to the effect that classical conjunction mean the same as the conjunction of a particular many-valued, or relevance, logic if the languages at issue are based on different semantical interpretations? Couldn't it be that a misunderstanding is lurking, as Quine surmised, and that the "deviant" logician is actually talking about a different concept, rather than disagreeing with the classical logician? I am inclined to think that pluralism can be turned into a more defensible view if some criterion is given – e.g. sameness of inferential behaviour – which allows to *compare* logical constants in deviant logics with their supposed classical counterparts while ruling out a change in their meanings. Of course, given the nature of their book, Beall and van Fraassen do not address such issues here (although I guess that Beall has a long story to recount about this in his forthcoming joint volume with Restall entitled *Logical Pluralism*[2]).

[1] Cambridge University Press, Cambridge, 2001.
[2] See http://vm.uconn.edu/~wwwphil/beall.html.

The first part of the book ("Preliminaries") is meant to provide the reader with the basic tools to follow the subsequent developments of the exposition. Chapter One introduces some of the motivation for going many-valued or modal. Chapter Two is a concise and highly readable primer on sets. Chapter Three carefully explains the basic syntactical and semantical notions connected with a logical system, with a welcome focus on the various forms of soundness and completeness. Chapter Four is an introduction to tableaux – and an excellent specimen of the authors' didactic abilities.

The second part ("Possibilities") is devoted to modal logic. Chapter Five is an introduction to the most remarkable normal modal logics. It is one of the best chapters of the whole book, where the strivings for clarity and completeness intersect at a well-chosen midpoint. Its distinctive feature is that each extension of **K** is motivated with reference to a particular philosophical interpretation of the modalities. Semantics are given in terms of Kripkean possible worlds models, while proof theory, as everywhere else in the book except for the fourth part, is given in terms of tableaux systems which highlight the connection with semantics.

Chapter Six explores what the authors call "variations on a theme". It begins with non-normal modal logics, that provide an excellent chance to illustrate a different interpretation of worlds (taken not only as possible worlds, or "ways things could be", but also as *im*possible worlds, or "ways things could not be"). Subsequently, we are offered a modal perspective on conditional logics and intuitionistic logic – where the latter heavily relies, once again, on Kripke's semantic interpretation.

The third part ("Paradox") is about many-valued logic. The choice of the title could sound somewhat surprising, because the connection between multivalued logics and paradox is far less evident than the obvious link "modal logic-possibilities". However, the *rationale* for this label soon becomes evident as Beall and van Fraassen opt to introduce many-valued logics mainly as sources of intuitions and techniques to defeat two time-honoured logical puzzles, the liar and the sorites.

Chapter Seven is devoted to the presentation of a simple example of many-valued logic, the system **FDE**. Here I have a small complaint. The authors seem to follow the established, but deplorable trend of conflating the meanings of the expressions "first degree entailment" and "tautological entailment". But this is not the way that Anderson and Belnap originally used these words in their book *Entailment*[3]. A first degree entailment is nothing but a single-premiss and single-conclusion sequent $A \Rightarrow B$, where A, B are formulae of a language that includes only classical connectives. No prescriptions concerning validity are contained in this notion. The logic of tautological entailments, on the other side, is a *particular* logic of first degree entailments, which singles out as valid just some of such sequents

[3] Princeton University Press, Princeton, 1975.

according to a specific criterion. Other logics of first degree entailments are certainly possible, as the authors themselves make clear in the subsequent chapter.

And so we have come to Chapter Eight, which is really effective. Three important 3-valued logics (corresponding to strong and weak Kleene logics and Priest's **LP**) are introduced and matched to just as many possible responses to the liar. The third logical value is successively interpreted either as "neither true nor false" (truth value gaps) or as "both true and false" (truth value gluts) or as "meaningless". The main problems with each approach are briefly but insightfully reviewed.

Chapter Nine, which is about vagueness and the sorites, appears to me slightly less accurate than the rest of the book. Quite understandably, since they are dealing with many-valued logic, Beall and van Fraassen focus on the supervaluational and the degree-theoretical approaches to the issue. However, the technical details of supervaluational semantics appear perhaps more complex than they need to be. Furthermore, the gist of the supervaluational diagnosis of the sorites does not emerge with sufficient force from the discussion provided here. As regards the continuum-valued approach, it must be noted that the solution to the sorites does not work in the version provided by the authors: if *all* the conditional premisses were less than absolutely true, the Lukasiewicz evaluation clause for implication would dictate that the first conclusion of the sorites ("99,999 grains make a heap") be already slightly less than absolutely true, which is counterintuitive – and, more than that, makes evaluation of the premisses too dependent upon where the starting point of the soritical sequence is located, which should be to some extent arbitrary (as far as a clearly positive case of possession of the relevant property is selected). It should therefore be remarked that, in this approach, only *some* of the conditional premisses must be evaluated as .99999 (or whatever), while the rest have to receive the value 1 corresponding to absolute truth.

The fourth part of the book, corresponding to Chapters Ten through Twelve, is devoted to metatheory. It contains a brilliant unified view of soundness and completeness theorems, as well as other metatheoretical results, for the previously encountered logics.

Apart from the minor criticisms that I raised here and there in this review, I am happy to say that *Possibilities and Paradox* is one of the books that I most enjoyed over the last years. One needs no soothsayer's ability to foretell that it is going to be a mandatory addition to any philosophical logician's library in the next future.